Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Probability

- Can one have a process which is at once a Markov provess and a martingale? What if the state space is assumed to be finite?
- The type of convergence in the Central Limit Theorem is the convergence in distribution. Why isn't the convergence almost sure?
  - HINT: What are the $\liminf$ and $\limsup$ of the values? Prove it.
- What is the difference between convergence in probability and convergence almost surely? [**Klass**]
- Give a sequence that converges in probability but not almost surely. [**Klass**]
- State the Weak Law of Large Numbers. [**Blackwell**]
- State and prove Kolmogorov's Law of Large Numbers. [**Blackwell**]
  - State a stronger Law of Large Numbers. [**Blackwell**]
- What general technique is used to prove the Strong Law of Large Numbers from the Weak Law of Large Numbers? [**Klass**]
- Give an example of a distribution on which the WLLN applies but the SLLN does not. [**Blackwell**]
- Write down the definition of a martingale and state a convergence theorem. [**Aldous**]
- Given a real-valued function F on $[0, 1)$ define functions f_n on $[0, 1)$ by setting, for $k2^n \leq x < (k + 1)2^n$,

  $$f_n(x) = F((k + 1)2^n) - F(k2^n)/2^n$$

  What does this have to do with convergence of martingales?
- What condition can be imposed on F so that f_n converges a.e. (according to the martingale convergence theorem)? What additional assumption will guarantee that $F(x) - F(0) = \int_0^x \lim f_n$?