Honors in Mathematics Writing a Senior Thesis (2021-2022)

1. Candidacy for Honors

A senior thesis is required for high or highest honors in Mathematics, where-as for straight honors (neither high nor highest), a senior thesis can be submit or four extra courses in Mathematics or approved related fields can be taken (above the required twelve courses). If a Math concentrator submit a senior thesis, then the Mathematics Department recommends that person for graduation with an appropriate level of honors (straight, high or highest or none) based on the course grades in mathematics, the senior thesis and a thesis examination. If a concentrator does not submit a senior thesis but takes the required four extra courses in Mathematics or approved related fields, then the Mathematics Department recommends that student for graduation with straight honors or no honors in Mathematics on the basis of the grades in all of the student's Mathematics and related field courses. The preceding rules are the only requirement for honors eligibility beyond the basic requirements of the Mathematics concentration.

The final award of Latin honors (*cum laude, magna cum laude or summa cum laude*) is made by the Faculty of Harvard University in accord with rules set forth in Handbook for Students, based on the Department's recommendation and a student's grades and subject to overall percentage quotas. (The university allows only a certain percentage of graduates to receive a given level of Latin honors. The Math Departments recommendations and the subsequent awarding of the English honors, highest honors, high honors and honors, is not subject to a percentage quota.)

Harvard University also awards degrees cum laude "on the basis of a student's overall record": that is, on the basis of grades in all subjects. There is no thesis requirement for this version of the cum laude degree. Under present faculty rules, people who qualify automatically receive their degrees cum laude if they do not receive honors in a field. Consult the Handbook for Students for a more detailed description of the general regulations relating to honors.

2. What is an honors thesis in mathematics?

An honors thesis in Mathematics is an original presentation of an area or subject in pure or applied mathematics culled from many sources in the published literature. The thesis can contain substantive, original mathematics, but most do not. Even with original mathematical results, a thesis must have a substantial expository component to be well received. Here is one way think about a thesis: Imagine giving a course on the subject to teach advanced undergraduates or beginning graduate students about the chosen topic. A well received thesis could constitute a set of lecture notes for that course.

Here is what a successful mathematics senior thesis is not: It is not a research paper done during the year or a previous summer or earlier. Even so, an account of completed research can be the sixth chapter in a six chapter thesis with the first five chapters being a detailed exposition of the subject matter and surrounding mathematics.

A thesis is also not a coauthored creation. The writing must be your own; although it is absolutely okay to ask your advisor and/or others to read drafts of your thesis and suggest improvements to both content and exposition. (You are encouraged to do this!) It is also absolutely okay to have a 'chapter six' in your thesis about research done in collaboration if the collaborators are appropriately acknowledged, and if the account of this research is your own account.

3. Why (or why not) write a senior thesis?

Here are four reasons for writing a senior thesis:

- The reading, writing and thinking about the chosen topic will take you towards the frontiers of current mathematical research.
- The thesis process provides a glimpse of life as a mathematics graduate student and as a professional mathematician.
- The process can lead to a close intellectual interaction with a professional mathematician (the thesis advisor).
- Almost all thesis writers felt that working on the thesis was a most challenging, confidence-raising, and fulfilling experience.

Meanwhile: Here are four reasons for not writing a senior thesis:

- Spend the last year exploring the myriad areas of knowledge that can be learned at Harvard.
- Spend the last year at Harvard working on a research project in mathematics or some other field.
- Spend the last year at Harvard working on a private sector project.
- Spend more time studying or preparing for a professional school.

Something to keep in mind: Whether you do or don't write a senior thesis has little bearing on success or not with job or graduate school or fellowship applications because the latter are due for the most part long before the thesis is due. In fact, it is likely that the only people who notice whether you did or didn't get honors will be your relatives (in which case, if you don't get honors, the Director of Undergraduate Studies will be delighted to tell them what a wonderful math person you are.)

4. The choice of a thesis topic

Any subject which makes substantive use of mathematics is suitable for a senior thesis whether the topic is in pure or applied mathematics (if in doubt about suitability, check with the Director of Undergraduate Studies). In general, focused theses on a circumscribed topic do well, whereas theses that present the elements of an over-arching theory tend not to. (Moreover, the latter are often less value to the writer.) Here is one more point to keep in mind: The quality of the presentation counts more than the sophistication of the topic. In particular, the thesis topic need not be 'cutting edge' mathematics, it only needs to be cutting edge mathematics to the thesis writer.

To find a thesis topic: First, choose a broad area of mathematics that interests you (i.e., analysis, geometry, topology, algebra, number theory, group theory, combinatorics, mathematical physics, logic, probability, ..., or some applied area of mathematics). Having chosen a broad area, then ask some faculty members who research that area for topic suggestions. Faculty will be delighted to give you many suggestions and to serve as your thesis advisor if you choose one. In this regard, don't be shy about asking for more choices if none strike your fancy. And, you can ask more than one person for suggestions simultaneously because there will be no hurt feelings if no topics are chosen. As a bonus, this strategy for finding a thesis topic finds you a thesis advisor too.

If the preceding strategy fails, then ask the Director of Undergraduate Studies in Math (or your Math faculty advisor) to help you find a thesis topic and advisor.

With regards to the preliminary step of finding a general area of research: Your courses, Math Department seminars and Math Table lectures can serve as sources to wet your interest in a general area of mathematics. Tutorials, reading courses and summer research experiences have also proved very useful in this regard.

And about timing: Best to start talking to faculty about senior theses topics in the semester before the start of your last year at Harvard. (This will be the spring of the third year for most people.) If you have a topic and advisor set up before your final year starts, then you can begin the thesis reading and research immediately at the start of your final academic year. And, if you have the time available, you can also do some preliminary reading before the start of that academic year. (This said, if you have compelling alternate activities set up for the summer before your last year–or for the winter break if you are finishing in the fall–then by all means make those a priority!)

5. The role of the thesis advisor

Almost all senior theses are written with a faculty advisor. The advisor's role is this:

- The advisor can point you towards the important literature on the chosen topic and to guide your understanding of the literature as you read and learn.
- The advisor can read early drafts of the thesis so as to
 - a) point out areas that need more exposition and/or elaboration,
 - b) point out errors in the presentation,

c) point out where the writing could be more elegant.

Take advantage of this opportunity for thesis mentoring by meeting regularly with your advisor (every two weeks is reasonable).

A professor from another department at Harvard or from another university can serve as the thesis advisor (this is not uncommon). However, if you do have a non-Math Department advisor, then you should arrange a second advisor in our department (a shadow advisor) who can suggest changes to your thesis early in the thesis writing stage so that the end result is in compliance with the standards of the Mathematics Department. The Director of Undergraduate Studies will help you find a shadow advisor when you start working on the thesis. To facilitate this, you should talk to the Director of Undergraduate Studies about your outside advisor plans very early in thesis process (at the beginning of the fall term for people graduating in May; in the beginning of January for those who are finishing in the fall).

Sadly, there have been on occasion students with *only* an outside advisor (or no advisor) whose thesis was not an acceptable mathematics thesis; and these people did not receive an honors degree. To avoid this unpleasant fate: If your primary advisor is not a member of the Harvard Math Department, you should submit a complete draft of your thesis to your shadow advisor no later than thirty days before the final due date for the senior thesis. This will give the shadow advisor the chance to suggest modifications that will put the thesis in compliance. (If you submit a draft to the shadow advisor at the last moment, then your shadow advisor won't have time to read it; and if the advisor does read it, then you won't have time to make any substantive changes.)

If you have no topic or no advisor or no shadow advisor by the beginning of your last year (September for May graduates, January for those finishing in the fall), see the Director of Undergraduate Studies for help finding a topic, advisor and/or shadow advisor.

As soon as you have decided upon your thesis advisor and the topic, notify the Undergraduate Studies Coordinator, <u>Cindy Jimenez</u> (room 334 in the Science Center).

6. Getting help

Choosing a thesis advisor and a topic can be a nerve-racking experience. Also, later on, you may encounter logistic or other difficulties while working on your thesis that your thesis advisor is ill equipped to deal with. In these circumstances please talk with the Director of Undergraduate Studies about your problems and worries.

7. Enrolling in Math 60r

You can free up time for thesis research by enrolling in Math 60r in one or both semesters of the last year at Harvard. This course is graded SAT/UNS. Students enrolled in Math 60r in their next to last semester (the fall for most people) need to submit a detailed thesis plan to the Director of Undergraduate Studies before the end of the fall reading period to receive a SAT grade. Students enrolled in Math 60r during their final semester at Harvard have to submit a senior thesis by the due date (see below for dates) in order to receive a SAT grade. If you change your mind about writing a senior thesis while enrolled in Math 60r, see the Director of Undergraduate Studies to change the course to a Math 91r reading course.

8. The thesis plan

Here is some advice: If you are writing a senior thesis, then submit a thesis plan in December to <u>Cindy Jimenez</u> (room 334 in the Science Center). This plan should be on the order of a page or two and it should contain a preliminary bibliography. If you are enrolled in Math 60r in the fall, the failure to submit such a thesis plan by the deadline (4pm on the last day of reading period) will result in an unsatisfactory grade for Math 60r. Whether enrolled in Math 60r or not, the process of producing a detailed plan will help focus your mind for the upcoming writing stage of the thesis in the final semester. There is an added benefit: The theses plans are reviewed by the Director of Undergraduate Studies to check on the suitability of the chosen topic and approach.

9. Format

No specific length is prescribed for a senior thesis. This said, theses exceeding fifty typed pages puts a strain on the reader if the writing is less than elegant. On the other hand, thesis below twenty or twenty-five pages are, for the most part, less than meaty and don't fare well as a consequence.

With regards to format: It is not necessary to have your senior thesis typed; a *legible* handwritten thesis is entirely acceptable. However, if you do want your thesis to look professionally typeset, LATEX is available on the FAS computer system. (LATEX and its TeX cousins are mathematical word processing programs.)

With regards to what to include: A bibliography *must* be included with your thesis. Also, please put your full name, e-mail address, telephone number, and your thesis advisor's name on the front page of the thesis.

Theses from previous years can be downloaded from this <u>website</u>; you can get an idea of length and style from these. But, keep in mind that theses on that website were not all Highest Honors theses.

10. The final deadline

If the spring term is your last semester: Two copies of the thesis must be handed in to the Undergraduate Studies Coordinator, Cindy Jimenez, in room 334 *no later than 4 pm on the first Monday after spring recess*.

If the fall term is your last semester: Cindy Jimenez must receive two copies of your thesis *no* later than 4 pm on the first Monday after the Thanksgiving break.

In no case is a late theses accepted. If your thesis is not finished by the stated deadline above, then hand in what you have completed up that point.

10. Hoopes Prize

Your advisor may nominate your thesis for a Hoopes prize if it is exceptionally well written. This is a Harvard College wide essay prize open to all Harvard senior theses. (The prize carries substantial monetary reward.) In the past, one or two Mathematics theses per year have received a Hoopes prize. Because this is an essay prize, even the most stratospheric thesis in mathematics will fail to win a Hoopes prize if its first pages (at the very minimum) are not written so that a non-mathematician (but scientifically literate individual) can understand it.

11. The Thesis Examination

Part of the evaluation process for the senior thesis is a thesis examination by the designated thesis reader; the exam is to determine whether you really understand what you wrote. To this end, the examination is will ask questions about statements in the thesis or about direct applications of the mathematics in the thesis. In particular, you will not get a strong honors recommendation if you don't fully understand what you wrote (including simple examples), or if your understanding is so narrow that you falter on questions that go slightly to one side of the chosen path your thesis took through the chosen subject area. This holds true no matter how impressively advanced your topic is or how elegant the writing.

With regards to the exam format: The thesis exam can be either oral or written (usually it is an oral exam). The format is up to the thesis reader. In any case, the exam lasts at most two hours (most less than that). For most people, it is reasonably gentle and low keyed.

With regards to the exam timing: A week or two after you submit your thesis, <u>Cindy Jimenez</u> will email you with the name of your thesis reader. Your reader will subsequently get in touch with you

(or you can get in touch with your reader) to schedule the thesis exam. The exams are typically scheduled before or, at latest, during the last week of classes in the relevant semester.

After your thesis examination, you can make an appointment to discuss your thesis and the examination with the thesis reader.

12. Honors Recommendations

Honors recommendations for May graduates are voted on by the faculty of the Mathematics Department at a meeting that takes place either at very end of April or during the first week of May. (For those finishing in the fall, the analogous meeting is at the beginning of January.) The quality of the thesis and the results of the thesis examinations have great weight in formulating the honors recommendations as do the grades in all mathematics and related field courses. This said, the weighting of each component cannot be quantified. This is to say that the decision on honors is not a matter of simply computing some sort of weighted average of thesis, thesis exam and math course grades; the process is inherently subjective, and there is really nothing that can be done about that. (Rigorous numerical formulae only hide the subjectivity, they in no way lessen it.)

Part of this inherent subjectivity involves the evaluation of the thesis. This is because different people will (unavoidably) have different points of view with regards to the difference between an honors thesis, a high honors thesis and a highest honors thesis. What follows are some general remarks about these differences.

- A straight honors thesis is a solid, workmanlike presentation that is free of substantive errors and omissions.
- A high honors thesis provides some real insight into the subject matter; and it does so via an elegant, clear and original of presentation.
- A highest honors thesis achieves the mastery of the subject and elegance of exposition on par with a review article by a professional mathematician.

Granted this subjectivity, there are also intangibles with regards to the relative weights given to the thesis, the thesis exam and the grades in mathematics and related field courses. For example, the distinction between a highest and high honors recommendations is often ambiguous: A highest honors recommendation requires neither a thesis with original theorems; nor extra course work, nor a record with mathematics graduate courses. By the same token, neither a thesis with original theorems, nor a straight A average, nor 18 mathematics courses (all at the graduate level), nor all three together guarantee a highest honors recommendation.

Recommendations from the departments are sent to the Faculty of Arts and Sciences for their deliberations. The Faculty of Arts and Sciences takes the recommendation into account when deciding the level of Latin honors at a meeting some days prior to commencement. (The possible Latin honor

distinctions are *cum laude*, *magna cum laude*, or *summa cum laude*.) These Latin honors are decided using criteria explained in the *Handbook for Students*. In practice, people recommended by the Department for honors or high honors almost always receive their degrees *cum laude or magna cum laude*, respectively, provided their grade-point average is above the relevant cut-offs set forth by the Faculty of Arts and Sciences in each year. About two-thirds of those recommended for highest honors graduate *summa cum laude*; the remainder receive their degrees magna cum laude or cum laude with highest honors in Mathematics.

13. Dates to Remember

- *Second and third years at Harvard*: Explore diverse areas of mathematics so as to ultimately choose a general area for the thesis.
- Last semester of the third year (the spring semester of the third year for most people): Ask several faculty members for specific suggestions of thesis topics, and then choose a topic. If you don't have an advisor or topic by the start of first days of your last year at Harvard, ask the Director of Undergraduate Studies to help you find them.
- The beginning of your fourth year at Harvard (the beginning of September for most people):
 - a) Notify <u>Cindy Jimenez</u> about your chosen advisor and topic.
 - b) If your advisor is not a member of the Mathematics Department, find a shadow advisor or ask the Director of Undergraduate to help you find one.
 - c) Enroll in Math 60r if you like.
- *Last day of the Reading Period in your second to last semester (December for most people)*: Submit a thesis plan to the <u>Cindy Jimenez</u> by 4 pm.
- Just before or at the start of your final semester (January for most people): Start writing if you haven't already!
- *March 1st of your last semester (but November 1 if the fall is your last semester)*: Submit a thesis draft to your advisor (and shadow advisor) for feedback and advice.
- *First Monday after spring break (but the first Monday after Thanksgiving break if the fall is your last semester)*: Submit two copies of the thesis to <u>Cindy Jimenez</u> by 4pm on that day.