## Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

## $C^{\ast}$ and von Neumann algebras

- What are Fredholm operators?
- What do they have to do with K-theory for operator algebras?
- Could you give some examples of interesting  $C^*$ -algebras with nontrivial K-theory?
- How does one recognize a compact operator? Give examples.
- Prove that the Hilbert-Schmidt integral operators are compact.
- One usually calls a  $C^*$ -algebra separable if it is represented on a separable Hilbert space. What are the  $C^*$ -algebras that are in fact separable as topological spaces?
- State Kaplansky's Density Theorem. [Jones]
- What is it good for? (e.g. in  $L^{\infty}(S^1)$ ) [Jones]
- Are the von Neumann algebras  $l^{\infty}(\mathbb{Z})$  and  $l^{\infty}(S^1)$  isomorphic? Can they be embedded in a  $II_1$  factor? [Jones]
- Define the index of a subfactor. [Jones]
- What are all the hyperfinite subfactors of index < 4? [Jones]
- Let S be the unilateral shift. What is the commutant of  $C^*(S^2)$ ?
- Do the Hilbert-Schmidt and trace class operators constitute  $C^*$  algebras under the Hilbert-Schmidt and trace norms, respectively?