Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Combinatorics and Combinatorial Algorithms

- State and prove Ramsey's Theorem, Hall's Theorem, Speiner's Theorem, Erdös-Ko-Rado Theorem, and the Erdös lower bound on Ramsey. [**Karp**]
- State Lovas local theorem and Alan's Theorem. [**Karp**]
- What is $R_1(n)$? [**Karp**]
- When is the Speiner bound tight? [**Karp**]
- What is the net work flow problem? [**Sinclair**]
- What is an algorithm to solve the problem? How do we know it terminates and what is a bound on the running time? [**Sinclair**]
- What is an algorithm with a better bound? [**Sinclair**]
- How can we use the algorithm to find a minimum cut? [**Sinclair**]
- What is a randomized algorithm for finding a minimal cut? [**Sinclair**]
- What is a bound on the error probability? [**Sinclair**]
- What does this tell us about how many minimal cuts there can be in a 1-graph? [**Sinclair**]
- What is an Eulerian poset? What is graded? What is a rank funciton? What is the length of a chain? What is μ of an interval? Why is it called Eulerian? [**Sinclair**]
- Consider monotonic paths from $(0,0)$ to (n,n) consisting of unit steps either $+(1,0)$ or $+(0,1)$. $\alpha \geq \gamma$ if α is never below γ. Define a hill to be a $+(0,1)$ step followed by a $+(1,0)$ step. Define a valley to be a $+(1,0)$ step followed by a $+(0,1)$ step. Given $\alpha \geq \beta$, define hills of α and valleys of β as good points. Define vallyes of α and hills of β as bad points. Show the number of good points is always greater than the number of bad points. [**Sinclair**]
- Talk about the Incidence Algebra on a poset. [**Klass**]
  - If we are to implement the Mobius inversion on the poset, do we need the functions in the Incidence Algebra to take values in a field? Does a ring suffice? [**Bergman**]
  - When is a function in the Incidence Algebra invertible? Prove it. [**Bergman**]
- Talk about the Mobius function for the product of two posets. Use it to describe the Mobius function on B_n, the Boolean poset of size n. [**Klass**]

- Prove that a finite meet-semilattice with 1 is a lattice. [**Bergman**]
- Is an infinite meet-semilattice with 1 necessarily a lattice? If not, find a counterexample. [**Bergman**]
- Prove that in a finite poset with a unique maximal element, that element is 1; find a counterexample in the infinite case. [**Bergman**]