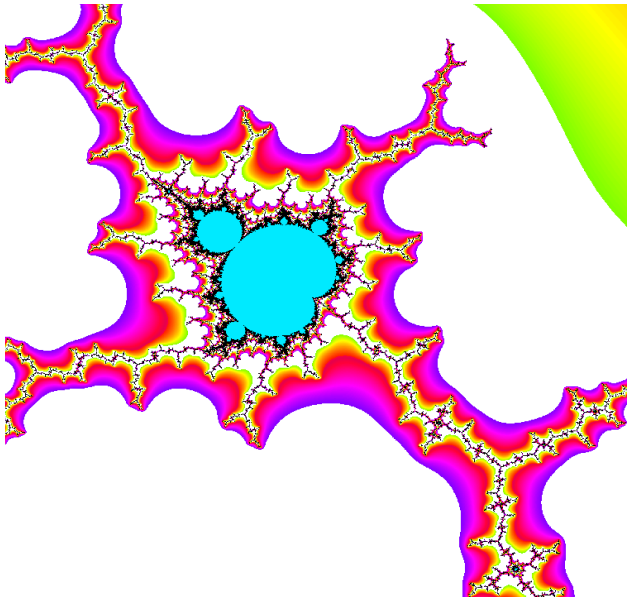
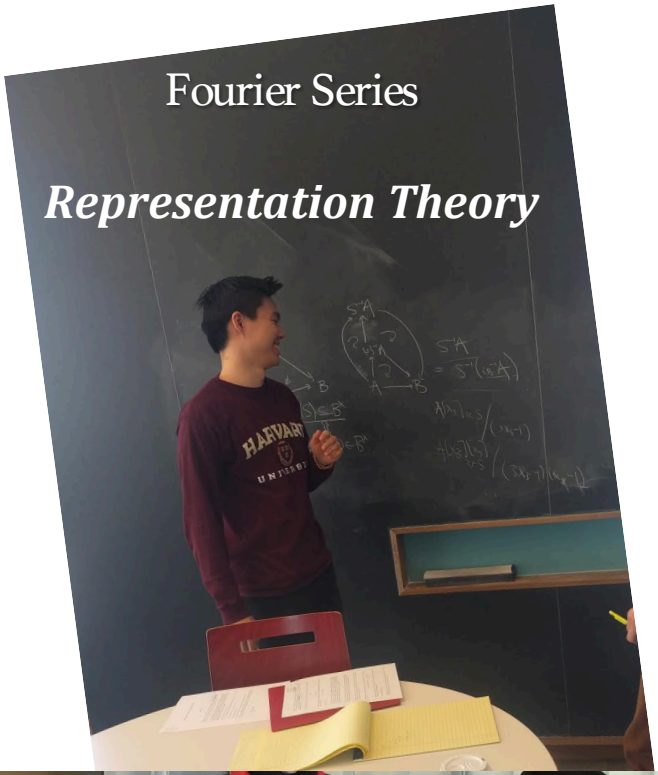


Tutorials

Category Theory

Arithmetic of Elliptic Curves

HARVARD UNIVERSITY DEPARTMENT OF MATHEMATICS UNDERGRADUATE PROGRAM



Math Table

Math Night

Welcome to the Mathematics Department!

This document is for you if you are considering becoming a concentrator in Mathematics, or if you have already declared and want more information. If you are considering the math concentration and simply want an overview, this first page might be enough for you. But keep reading if you'd like to know more.

Why Mathematics? What Can You Do With Math?

Mathematics is the science of order, and mathematicians seek to identify instances of order and to formulate and understand concepts that enable us to perceive order in complicated situations.

Concentration in mathematics is an excellent preparation for a mathematics career in either academia, science, industry, secondary school education, finance, law or government. Most advanced disciplines rely on mathematics. For example, the physical sciences, life sciences, computer science, economics, data science and the social sciences are all mathematically intensive fields. As a consequence, many mathematics concentrators take jobs after Harvard in industry, education, finance, medicine or other professions that are not "classically" mathematics. Others have gone on to professional training in law, business, medicine or actuarial sciences. Roughly a third of our concentrators go on to pursue a graduate degree in mathematics or a closely related field such as computer science or statistics.

A Harvard degree with a concentration in Mathematics opens just about any professional door. The mathematics concentration is also very flexible with regards to electives, allowing you to pursue multiple interests during your time at Harvard.

Concentrators enjoy a department lunch with Professor Elkies.

Overview of Requirements and Possibilities

Concentration in mathematics requires a minimum of twelve letter-graded course of which at least eight must be courses labeled as Mathematics. The remaining four can be either labeled Mathematics or courses in officially approved *related fields*. Of the eight (or more) courses labeled as Mathematics, at least one must be a course in analysis, one must be a course in algebra, and one must be a course in geometry/topology.

Students are also required to fulfill the Math Expository Requirement, by submitting a 5-page expository paper on some approved mathematics subject by the end of the third year at Harvard.

A candidate for high or highest honors in Mathematics must submit a senior thesis. The thesis can be on any topic in pure or applied mathematics that is not directly covered in your course work. It should be an original exposition of material culled from multiple sources. Note that most theses do not contain new "theorems." A student who wishes to be considered for straight honors (neither high nor highest) can either submit a senior thesis or take four extra courses (above the required twelve courses) in Mathematics or approved related fields.

Some students participate in the AB-AM program in their fourth year at Harvard to graduate with a Master's Degree in Mathematics.

Joint concentrations with other departments are frequent, with Mathematics as either the primary or allied field. Also, math concentrators can earn teaching certification with the Undergraduate Teachers Education Program (UTEP). The Mathematics Department offers a "Mathematics and Teaching" option for students enrolled in UTEP. If you are tangentially interested in Mathematics, consider a secondary field in Mathematical Sciences. The requirements are relatively broad and not difficult to fit in with another concentration.

So many options! To know more, read on.

What do I need to know for my first few semesters at Harvard?

Keep in mind: You will need to learn to read and write mathematics proofs in order to benefit from higher level mathematics courses. You should learn to do this by the end of your second year. Many courses directly teach this skill: any or all of Math 101, 112, 121, 130 if you take Math 21; or one of Math 22, 23, 25 if you don't take Math 21.

Let's start with your math placement test: please do try to follow it. This table explains what we suggest you take, depending on your placement. If you want to try something different, or if you have any questions or doubts about your placement, please reach out: our contact info is on the last page.

For details on courses numbered 18 to 101: `https://www.math.harvard.edu/media/Beyond-2020.pdf`

Placement	What you should take first	What you can take next
Math M, 1	Start with your placement, and take up to and including 1b.	Take 21a,b. Concurrently, take **Math 101**, or see next row of this table.
Math 18, 19, 21	If you are not sure you have a strong interest in math, but want to keep more options open: take 21a,b, and consider taking **Math 101** concurrently. If you have already started or taken Math 18 or 19, consider **Math 101** and come in to talk to us.	Consider any combination of: **Math 101**; Math 112 (real analysis); Math 121 (linear algebra); Math 130 (classical geometry). These teach proof-writing skills. Also accessible: Math 152 (discrete mathematics) and Math 157 (Math in the World).
22, 23, 25	If you have a strong interest in math, start with 23a,b or 25a,b.	Math 122 (1st course in abstract algebra). After (or with) 122, consider Math 131 (topology) or Math 114 (measure, integration and banach spaces).
or 55	For a fast-paced, challenging course that covers more topics more deeply than 23 or 25, take Math 55a,b.	Math 114, 131, 123, 132.

Math 101 is strongly suggested (if not taking 22, 23, 25 or 55). It offers a taste of higher mathematics, introduces you to proofs, and also to the main branches of mathematics – a great way to gauge whether you are interested in the math concentration!

You can find more information about "how to structure a good program" in the section of the same name of this pamphlet: `https://www.math.harvard.edu/undergraduate/courses-and-concentration-information/`. This pamphlet also has a section containing sample programs, depending on your placement and interests.

For information on placement test scores and calculus courses, we have special placement advising office hours at the beginning of each semester, see here: `https://www.math.harvard.edu/first-year/`.

"I was scared about taking my first class that required writing proofs. Everyone is. But you quickly realize that writing proofs is just formalizing the understanding that you have. So it just comes naturally as you're learning." Professor Eli Grigsby was a math concentrator at Harvard and is now tenured at Boston College. Full interview: `http://legacy-www.math.harvard.edu/preceptor/videos/2015/`

Requirements for the Mathematics Concentration

It is important to get to know the three main branches of mathematics: analysis, algebra and geometry. More advanced material often reaches across these branches, and tools and intuition from each branch might be useful in another. **Because of this, the department requires you take at least one course in each one of the three main branches (analysis, algebra, geometry) for your concentration.**

This *breadth requirement* applies to joint concentrators, but not if you are doing a secondary field in the mathematical sciences. Note that Math 101 can count as a course towards one of these branches for the math and teaching option. We still encourage you to sample as many of these branches as you wish!

Analysis The concept of function provides us with the means to study dependence and change. The study of functions, often in connection with the limit concept, is called analysis. The most effective tool for this study is the infinitesimal calculus that analyzes the relation between functions and their derivatives.

Math 110	Vector Space Methods for Differential Equations
Math 112	Introductory Real Analysis
Math 113	Analysis I: Complex Function Theory
Math 114	Measure, Integration and Banach spaces
Math 115	Methods of Analysis
Math 116	Real Analysis, Convexity, and Optimization
Math 117	Probability, Random Processes, Economic Applications
Math 118r	Dynamical Systems

Core courses are boxed in purple: these teach basic skills needed for a lot of higher math. Not required, but strongly suggested.

Algebra The study of number systems and their generalizations is called algebra. Here the primary concepts are group, ring, field, and module.

Math 121	Linear Algebra and Applications
Math 122	Algebra I: Theory of Groups and Vector Spaces
Math 123	Algebra II: Theory of Rings and Fields
Math 124	Number Theory
Math 129	Number Fields

Students may cross-register for courses at MIT. Get prior permission from the math DUS if you want such a course to count for concentration credit.

Geometry/Topology Geometry goes far beyond the classical study of the space we live in to include spaces of high dimension and topology, the abstract theory of shape.

Math 130	Classical Geometry
Math 131	Topology I: Topological Spaces and the Fundamental Group
Math 132	Topology II: Smooth Manifolds
Math 136	Differential Geometry
Math 137	Algebraic Geometry

Courses offered are subject to change. The course catalog contains our full offerings.

Other Other important subjects of mathematics do not fall within the three main branches of mathematics but are worthwhile to study.

Math 141,145	Logic courses
Math 153	Mathematical Biology and Evolutionary Dynamics
Math 154	Probability Theory
Math 155r	Combinatorics
Math 157	Mathematics in the World

Not sure about Math yet? Math 121, 122 and 112 provide useful tools for many other disciplines.

Math 91r is a reading course for students who want to learn a particular topic not covered in a regular course or tutorial. Details: `https://www.math.harvard.edu/media/Courses-2020.pdf`

Requirements for the Mathematics Concentration (continued)

Course Requirements

- 8 letter-graded courses in Mathematics (Mathematics Ma and Mb together count as one course of concentration credit), not cross-listed, at least 4 of which must be at the 100 level and satisfy the breadth requirement mentioned on the previous page.
- 4 letter-graded courses in either Mathematics or a related subject. Numerous courses in Applied Mathematics, Computer Science, Economics, Engineering Sciences, Physics, Applied Physics and Statistics count towards this requirement. Other departments also have a handful of approved courses. The Student Handbook contains all the details.
- A 5-page expository paper in mathematics, written in second year or before the end of the third year reading period, under the supervision of a professor or tutor in a tutorial (Math 99r), a reading course (Math 91r), or a 100- or 200-level course the student is enrolled in. Students enrolled in a tutorial satisfy this requirement as part of the tutorial.

Tutorials are encouraged because they are taught in a small class by a graduate student – a completely different experience from typical classes!

One Freshman Seminar can be substituted for one of the twelve letter-graded courses listed above, provided: it is taught by Department of Mathematics faculty; it is not used to satisfy the breadth requirement; permission is obtained from the Director of Undergraduate Studies before the approval of the Plan of Study.

Requirements for Honors Mathematics Concentration

A candidate for high or highest honors in Mathematics must submit a senior thesis in addition to the other requirements. The thesis should be an original *exposition* of material in pure or applied mathematics culled from several sources. Note that it need not contain original mathematical research. A student who wishes to be considered for straight honors (neither high nor highest) can either submit a senior thesis or take four extra courses (above the required twelve courses) in Mathematics or approved related fields.

The department strongly urges concentrators to write a senior thesis; this experience provides a much better glimpse of mathematical research and graduate work than taking courses, and is also great for students who prefer independent learning. More about senior theses can be found in the pamphlet "Honors in Mathematics": `https://www.math.harvard.edu/media/Honors-2020.pdf`. You can take Math 60r in your senior year if you are writing your senior thesis.

Concentration Advising

The department assigns all concentrators a faculty member as their concentration advisor. Your advisor can help you plan your coursework, and will sign your Crimson Cart. How much contact you have with your advisor and how helpful they are will depend almost entirely on your initiative. Drop by during their office hours or during our 4pm tea and cookies (see under *Activities*), or invite your advisor to lunch at your House. Your advisor can also help you choose a thesis topic, a thesis advisor, learn about mathematical research, and apply to graduate school. If you would like to change your assignment, please talk to Cindy Jimenez in SC 334 (cindy@math).

The department also has every third year student come in for a scheduled 20 minute advising session with two faculty members in the spring semester; post-graduate options and senior theses are often discussed. Third year students will receive an invitation letter by email in the spring from Cindy Jimenez.

We offer the AB-AM degree, for Math concentrators to obtain a Masters degree (AM) in Mathematics in addition to their Bachelors degree (AB) during their four years at Harvard. For details, see the corresponding pamphlet: `https://www.math.harvard.edu/media/AB-AM-2020.pdf`

Combining Mathematics With Another Discipline

There are many ways to combine your mathematical interests with other academics.

Joint Concentrations

Joint concentrations with other departments are not only possible but very common. Joint concentrations are honors only. If Math is the primary field, the student must fulfill the requirements for Honors in Math (see previous page) plus whatever other requirements are called for by the allied department. If Math is the allied field, the Math requirements are as follows: five letter-graded courses in Mathematics with a grade of C− or higher and averaging at least B+; at least three of those courses must be at the 100-level and must satisfy the breadth requirement (see previous page); no senior thesis required; no expository paper required either.

Mathematics and Teaching Option

Math concentrators who would like to earn teaching certification to teach in Massachusetts (and the 40+ states with which Massachusetts has reciprocity) public schools after graduation may want to look into the Undergraduate Teachers Education Program (UTEP): `http://utep.gse.harvard.edu/`. The Mathematics Department offers a "Mathematics and Teaching" option for those students concurrently enrolled in UTEP. Students choosing this option need to fulfill slightly different course requirements. In particular, they take required UTEP and Graduate School of Education courses, but need to fulfill slightly less of the math concentration requirements. Details are in the Student Handbook.

Secondary Field in the Mathematical Sciences

If you'd rather concentrate in another discipline, but still take math or "mathy" courses and be recognized for it, you can do a secondary field! Requirements are four courses (for a letter grade, grade of C- or above) in mathematics, applied mathematics or statistics, of which at most two can be in statistics. The math and applied math courses must be numbered 104 or higher; statistics courses must be numbered 110 or higher. Details are in the Student Handbook.

"I chose to study math because of the extraordinary interconnectivity of the subject. The theory goes very deep, and the farther you go in your studies, the more connections you draw. I love the way that things seem to piece together nicely in mathematics, and the things you prove are true will always be true. Studying math takes a lot of patience, and I think the first year is especially hard, depending on one's experience of math in high school. If you are considering math, make sure to think give the problems time. The more you do math, the easier approaching these problems will be, even if the material gets more difficult."

Kat Zhou, Mathematics, Class of 2017

"Exploring math allows you to really understand how all the sciences work and appreciate the beauty of what humans have discovered."

Vikram Sundar, Mathematics, Class of 2017

"Math is beautiful! From simple principles one can deduce a broad range of interesting facts using nothing more than the trappings of logic. Math consistently challenges me to think hard, solve problems, and learn exciting theories that really stretch the imagination!"

Jeffrey Ling, Computer Science and Math, Class of 2017

"The problem-solving techniques you learn in math are incredibly helpful in other disciplines. Studying math has made me a better computer scientist, statistician, and thinker."

Mark Yao, Mathematics, Class of 2016

"Math is so useful, teaches you to think logically and rigorously, is highly rewarding and transferable, makes a great resume, and allows you to take so many electives."

Jeffery Durand, Mathematics, Class of 2017

Activities

The Harvard Math Department prides itself on its informal atmosphere. Come to these events regularly, and get to know other concentrators, the faculty, and graduate students.

Due to Covid-19 restrictions, in person activities will be absent for at least the fall 2020 semester

Math Department Welcome Party
At the beginning of the fall semester the Math Department holds a party in the Austine & Chilton McDonnell Common Room on the 4th floor of the Science Center. Please come and meet other math concentrators, faculty members, math graduate students, and our staff. Refreshments provided.

Math Night
During Math Night you can find friends from your math class, work on problem sets and get help from your course assistant. Extra special food is served! In Leverett House Dining Hall, Mondays 8-10pm.

Second Year Studentss Welcome Party
Every spring the math department holds an informal meal to welcome second year students who have declared math as their concentration. A few upper-year students and faculty are present so you can ask them about the concentration.

Women in Math
This group hosts monthly lunches for women-identified members of the department, including concentrators. We try to add new female concentrators to this mailing list as they come in, but please contact us if you would like to be added.

Math Competitions
Each year undergraduates across the nation compete in the Putnam Math Competition, an examination given in two 3-hour sessions in early December. There are substantial prizes. Sign up to participate in late September on the bulletin board opposite room 323. Other fantastic competitions are the team-based Mathematical Contest in Modeling (MCM) and the Interdisciplinary Contest in Modeling (ICM) that occur in late January. Email Rosalie Belanger-Rioux (rbr@math) to sign up.

Math Department Luncheons
Held roughly every two weeks, these department-wide catered lunches occur in the common room, and are open to all members of the department, including concentrators. Please join us!

Gender Inclusivity in Math student group
The Gender Inclusivity in Math (GIIM) student group is dedicated to creating a community of mathematicians supportive of women interested in math. They have social events, a speaker series, a fantastic conference, a great advising pamphlet, and more: `https://harvardgiim.org/`.

Harvard Undergraduate Math Association
The Harvard Undergraduate Math Association (HUMA) is a student group that organizes many events (board games, movies, faculty dinners, Putnam practice, etc), open to anyone who likes math. Find out more here: `http://harvardhuma.org/`.

Math Table
The Math Table meets Tuesdays 5:30pm in room SC507 in the Science Center over dinner to hear talks, mostly by undergraduates. Anyone, at any level of math literacy, is welcome. Talks generally cover topics which fall outside the scope of regular courses. The talks serve three purposes: to introduce undergraduates to various fields of math, to suggest topics for a senior thesis, and to provide a forum for undergraduates to deliver math talks. The Rogers Prizes are awarded yearly for the two best talks delivered by undergraduates. Details here: `http://math.harvard.edu/mathtable/`.

The Common Room
The Austine & Chilton McDonnell Common Room (4^{th} floor of the Science Center) is open to math concentrators and friends of the Math Department. Tea and cookies are served at 4pm Monday through Wednesday (occasionally on Thursday). Cheese, bread, and juice is offered at 4:15 or so on Fridays.

The Mathematics Library
The Birkhoff Mathematics Library on the third floor of the Science Center holds many mathematical books and journals. Textbooks used in math courses are held on reserve there. It is a noncirculating library; its resources may be used only on the premises. Open Monday through Friday 9am-5pm.

Math concentrators may request card access to the department from the Main Office, SC 325.

Jobs, Internships, Study Abroad

Course Assisting
The Math Department hires undergraduates as course assistants (CAs). A CA grades homework assignments, leads a section once a week, holds office hours and attends the lectures. Hiring for these positions takes place 1-2 months before each term. For more information on CA positions (for the calculus courses as well as the upper-level courses), go to: `http://math.harvard.edu/cas/`.

Study Abroad
The Office of Career Services can help you arrange to spend a semester or a year studying abroad. Consider the well-regarded "Budapest Semesters in Mathematics" and the newer "Budapest Semesters in Mathematics Education"; both programs are held in Hungary. The programs allow students to spend one or two semesters respectively studying combinatorics, algebra, and discrete math, or studying the teaching of mathematics using a student-centered approach, in small groups with senior Hungarian mathematicians. All classes are taught in English. For more information visit `https://www.budapestsemesters.com/` or `https://bsmeducation.com/`.

Another great program is "Math in Moscow", `https://mathinmoscow.org/`.

Summer Research
There are opportunities to do pure or applied math during the summer. Browse thre the undergraduate bulletin boards near rooms 320 and 503.

The Harvard Office of Undergraduate Research and Fellowships also hosts residential research programs over the summer. PRISE is probably the one that aligns most with mathematical interests. `http://uraf.harvard.edu/summer-residential-research-programs`

The National Science Foundation (NSF) sponsors a number of Research Experiences for Undergraduates (REU) programs in mathematics every year. You may wind up learning something interesting, *and* making money! Info here: `http://www.ams.org/programs/students/emp-reu`

Large companies often hire undergraduates to do research (usually somewhat applied math) during the summer. Reach out directly to the company very early (December is not too early) and look for advertisements posted on the Division of Engineering and Applied Sciences' bulletin boards.

The math department usually holds an info session on summer research in early December. More info here: `http://math.harvard.edu/undergrad/research.html`. Also, check with the Harvard Office of Career Services.

Interested in graduate school? Information is available in the pamphlet "Graduate Schools and Fellowships in Mathematics" at `https://www.math.harvard.edu/media/GradSchool-2020.pdf`.

A Women in Math lunch with undergraduates, graduate students, post-docs and faculty. See "Activities" page for details.

Contact us

- Interested in math? Forward your email address, name and year at Harvard to Cindy Jimenez in order to receive information about math-related events of general interest.
- Advising Fortnight: the department hosts an "open house" in the common room with refreshments, and also special walk-in office hours. More info: `http://advisingfortnight.fas.harvard.edu/`
- Information about tutorials, jobs, graduate schools, fellowships, and other matters is posted on the Undergraduate Bulletin Boards opposite rooms 320, 427 and 503.
- The Monthly Math Memo is a newsletter sent to math concentrators.
- Information sessions on writing a senior thesis (one in the fall for fourth year students writing a thesis and one in the spring for second or third year students who want to know more about writing a thesis) happen every year. More information can be found here: `https://www.math.harvard.edu/media/advice-senior-thesis.pdf`.
- Advice on obtaining letters of recommendation: `http://people.math.harvard.edu/~hirolee/rec-letters`.

"Will the rest of math be like this course? This course was great!!" Anthony Clark '16 was a math concentrator. You can find the full interview, including lots of advice for students of math, here: `https://youtu.be/1HfSBL1deac`

The department hosts a welcome lunch for new sophomore concentrators every year after they declare concentration. A few upper-year students and faculty are also invited, to share advice and answer questions.

Professor Cliff Taubes
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Science Center 504
chtaubes@math.harvard.edu

Senior Lecturer Wes Cain
Assistant Director
of Undergraduate Studies
Science Center 515
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Harvard Mathematics Department website: `https://www.math.harvard.edu`
Harvard College Handbook Math Concentration: `https://handbook.fas.harvard.edu/book/mathematics`