# Math for First Year Students

The following chart lists the introductory mathematics courses.

<table>
<thead>
<tr>
<th>MathMa → MathMb → Math 1b →</th>
<th>Math 18a ↘</th>
<th>Math 18b/19b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 19a</td>
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<tr>
<td>Math 21a → Math 21b</td>
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<tr>
<td>Math 22a → Math 22b</td>
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<td>→ 100 level</td>
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<td>Math 23a → Math 21b,c</td>
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<td>Math 25a → Math 25b</td>
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<tr>
<td>Math 55a → Math 55b</td>
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<tr>
<td>AM 22a</td>
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<td>AM 22b</td>
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</tbody>
</table>

Each student receives a math placement recommendation that indicates either Math Ma or Math 1a or Math 1b or the column with Math 18-55 and AM22a. More is said about all of the courses in the diagram and the math placement recommendation in what follows. Given the starting course, the arrows indicate the most likely next course (or suite of possible courses). Except for Math 21a and Math 21b, the column with Math 18a at the top indicates courses taught only in the fall semester and the next column to the right (with Math 18b/19b at the top) indicates courses taught only in the spring semester. Math 21a,b are taught in the fall and again in the spring.

## 1. The crucial information
- **Placement:** The placement recommendation must be taken very seriously. A student who questions the placement recommendation should talk to one of the **math placement advisors.** The advisors can sort out where the student truly belongs. The placement advisor schedules and contact information can be found at [http://www.math.harvard.edu/sectioning](http://www.math.harvard.edu/sectioning) (Please wait until August). Do not take placement advice from other students or alumni because the risk of error is too large. There is also a special video by the Math Department placement advisors to tell you about the meaning of their placement scores. The video is called *Making sense of your placement scores* and can be found at the website [https://youtu.be/wVk0T5JyQuY](https://youtu.be/wVk0T5JyQuY). The chart on the next page explains the logic behind the math placement recommendations.
The placement test does not distinguish between the courses in the Math 18, 19, 21, 22, 23, 25, 55 and AM 22 column. What a student placed in that column takes depends partly on the student’s interests and partly on how much time the student wishes to spend on mathematics this year. Any and all questions about which course to take in this column will be answered at a special Zoom meeting in mid August titled Shall I take Math 18, 19, 21, 22, 23, 55 or AM22? The date is Saturday, August 15 from 11am to 1pm. The meeting url is https://harvard.zoom.us/j/97664431413. Please don’t miss this meeting if your placement recommendation is in the Math 18-55, AM 22 column.

**Sectioning:** The teaching in Math Ma, 1a, 1b, 21a and 21b is done in small, equivalent sections. Each student must register for the course and then request a section. Each section meets on Monday, Wednesday and Friday. As for the time of day: This is where the request comes in: There are multiple times to choose from and each student must request a time. Information for how to request a section time can be found on the course websites. Official section assignments will be emailed to students in the last week of August and the sections
start meeting in the first week of September. Math 23a is taught in two sections; but these are not equivalent (see the course description below and the catalogue description or course website). The other courses on the list are not taught in sections.

Placement recommendation questions: There are lots of advisors ready to answer questions about Harvard Math Placement recommendation and to help each student choose the correct starting math classes. The math placement advising schedule and contact information can be found online at http://www.math.harvard.edu/sectioning. (Placement advising won’t be available before this coming August 10.)

Questions about higher level math courses: Questions about math courses numbered above 55 can be addressed to Prof. Cliff Taubes (chteubes@math.harvard.edu). A student should make an appointment to talk with him. Questions can also be addressed to Wes Cain (jcain2@math.harvard.edu).

2. Brief course descriptions

The following briefly describes the courses for first year students:

- **Math Ma,b**: This is a two semester course which combines pre-calculus with one variable calculus including the basics of integration and differentiation. A student who completes this sequence enters directly to Math 1b. This course satisfy the QRD requirement.
  * ESPa,b: This is an academic enrichment program for people at the Math Ma,b level interested in fields of science, technology, engineering or medicine. See https://emergingscholars.math.harvard.edu for more information and for how to apply (or contact the Harvard Math Department). A Zoom meeting for students about the ESP program will be held in mid August titled the The Emerging Scholar Program information session.
- **Math 1a**: This is a standard first semester one variable calculus course that covers differentiation and the basics of integration. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).
- **Math 1b** is the second semester of a basic introduction to calculus sequence. However, be aware that Math 1b has a tripartite curriculum: It teaches integration of densities over surfaces and volumes, it discusses sequences and series, and it gives an introduction to ordinary differential equations up to second order equations with constant coefficients. All three parts of the curriculum are important for natural and social science concentrations. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).
- **Math 18a**: This course is taught in the fall only. Math 18a covers the concepts and techniques of multivariable calculus most useful to those studying the social sciences, particularly economics. Math 18a should not be taken in addition to Math 21a, but Math
Math 18/19 b or Math 21b may be taken before or after Math 18a. (Math 18a and 21a will be together for a some weeks before splitting.) This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).

- **Math 19a:** Math 19a is given in just the fall; it teaches multivariable calculus and differential equations for applications to the life sciences. It is recommended by those taking the new Life Science 1a,b courses and by the life science concentrations (this means Biological Anthropology, Chemical and Physical Biology, Human Evolutionary Biology, Molecular and Cellular Biology, Neurobiology, Organismic and Evolutionary Biology, and Social and Cognitive Neuroscience.) In any event, it is usually preferable to Math 21a for those majoring in a life science except for students who plan to take Physics 15/16. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).

- **Math 18b/19b:** This one course is given only in the spring semester. Math 18b/19b is a follow-on course for both Math 18a and Math 91a. It is for people interested in life sciences or social sciences or economics. The course teaches linear algebra with enough probability and statistics to forgo the beginning statistics courses such as Stat 100, 102, 104. Some programming will also be taught. All of the linear algebra in Math 21b is taught; The differential equations in Math 21b is traded for probability and statistics. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).

- **Math 21a:** This course covers the basics of multivariable calculus in two and three dimensions: Curves and surfaces, functions and their derivatives, the calculus of variations, multi-variable integration, integration on curves and surfaces, multivariable generalizations of the fundamental theorem of calculus. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).

- **Math 21b:** This course covers the basics of linear algebra in dimensions 2, 3 and higher. A good part of the course uses linear algebra to study ordinary and partial differential equations. This course should satisfy the QRD requirement (check the Course Catalogue in August for confirmation).

- **Math 22a,b:** This course covers multivariable calculus and linear algebra for students interested in theoretical sciences. It covers the same topics as Math 21a,b but with more rigor. Students are taught techniques of proof and mathematical reasoning. The workload and content is comparable with the 21 sequence. But unlike in the later, the linear algebra and calculus is more interlinked.

- **Math 23a,b,c:** This sequence likewise treats multivariable calculus and linear algebra in a rigorous, proof-oriented way. There are two alternatives for Math 23a. The Friday meeting pattern is taught in a format where students watch videos before class and then do presentations and problem solving in class. It covers elementary single-variable real analysis, which may have been omitted from an AP Calculus course in 2020, but does satisfy the QRD requirement. The MWF meeting pattern has interactive lectures, makes extensive use
of computational tools (R scripts), and satisfy the QRD requirement. It assumes some prior experience with sequences and series, continuity and limits.

Students can take either Math 23b or Math 23c in the spring. Math 23b is geared toward physics and does not satisfy the QRD requirement. Math 23c is geared towards data science applications and does satisfy the QRD requirement. Either 23b or 23c may be taken after either meeting pattern of 23a, and either will provide full preparation for courses that require linear algebra and multivariable calculus. This course is typically more time consuming than Math 21a,b.

- **Math 25 and 55:** These are theory courses that should be elected only by those students who have a particular interest in and enjoyment of abstract mathematics, as well as a solid understanding of one-variable calculus. These courses assume a willingness to think rigorously and abstractly about mathematics, and to work hard. Both courses study multivariable calculus and linear algebra plus many very deep related topics. These courses come with an iron clad guarantee that you will be challenged by the mathematics.

  * **CHOOSING BETWEEN MATH 22, 23, 25 OR 55:** Math 25 differs from Math 22 and Mathematics 23 by virtue of the work load in Mathematics 25 being significantly more than in than in these other courses; but then Mathematics 25 covers more material. Note however that any course that asks for Math 25 as a prerequisite accepts Math 23 and Math 22 as well. As noted above, Math 23 differs from Math 22 in the way it is taught, in the workload and, to some extent in the focus. Meanwhile, Mathematics 55 differs from Mathematics 25 in that the former assumes an extremely strong proof oriented mathematics experience. Mathematics 55 requires the consent of the instructor.

- **Applied Math 22:** The Applied Math 22a,b sequence (*Solving and optimizing* and then *Integrating and approximating*) covers most of the same material as Math 21a,b but the order of presentation is different with linear algebra appearing in AM 22a and much of multivariable calculus in AM 22b. It is also taught from a somewhat more applied point of view and with less on differential equations. AM 22a,b are taught in large lecture format.

**NOTE:** Math 18 or 19a and Math 18/19b can be taken in either order; Math 21a,b can also be taken in either order. Math 21a can be taken with Math 18/19b and Math 19a can be taken with Math 21b. Math 22b can be taken after Math 21a and Math 21b after Math 22a.

**OTHER COURSES:**

- **Math 101:** Math 101 (fall and spring semesters) is designed to give people with a Math 1b level background and with interest in mathematics a taste of what modern mathematics is all about. This course can be taken concurrently with Math 21a or 21b. It is not to be taken with Math 23, 25 or 55 (without special permission). It can be taken with Math 22 (concurrently or after). Math 101 also gives a good background for writing and following mathematical proofs. This skill will be needed in any event for most higher level math courses. This skill is also taught in Math 22, 23 and Math 25, and in a few other 100 level courses.
• **Freshman taking other 100 level courses:** This is OK in principle (but concurrent registration in Math 25 or 55 is recommended). In any event, any freshman that is considering this has to talk first with the Mathematics DUS, Professor Cliff Taubes (chtaubes@math.harvard.edu).

• **Courses at MIT:** Any freshman considering a mathematics course at MIT must talk first with Professor Taubes.

• **Freshman Seminars:** Most freshman seminars by Mathematics Department faculty members can be used as one of the 8 required Mathematics courses for a concentration in Mathematics. To do this, a student must petition to Professor Taubes after taking the course. Freshman seminars are to be recommended. There is one this fall taught by Professor Hugh Woodin.