Courses in Mathematics (2024-2025)

This document gives a brief description of the various courses in calculus and some of the intermediate level courses in mathematics; and it provides advice and pointers for planning your course selections. If you are a Mathematics Concentrator, or are considering entering the Mathematics Concentration, and if you are seeking some overview of the courses and how they fit together, then this document might be useful for you. Even so, make note that the guidelines presented below are only guidelines. Keep them in mind when you are deciding how to structure your program, but be sure to talk to your advisor in the Mathematics Department or to the Director of Undergraduate Studies before you submit your Crimson Cart at the start of the semester.

1. Calculus

Math 1a/b is the standard first-year calculus sequence. If you are thinking about concentrating in math and have not taken calculus before, take Math 1a,b as soon as possible (take Math Ma,b first if you are not yet ready for Math 1a,b). If you have had a year of calculus in high school, and if you have passed the Advanced Placement examination in BC Calculus with a score of 4 or better, then you may be advised to begin with Math 21 a,b which is the next level sequence (multivariable calculus in Math 21a and linear algebra in Math 21b).

If you scored a 5 on the BC Calculus exam and if you are advised to take Math 21a,b, then you can consider taking the respective Math 22 or Math 25 or 55 courses instead of Math 21. Math 25 and 55 are intense but very rewarding courses, and both 25 and 55 require extensive work outside the classroom. To succeed in the latter two, you must like doing mathematics for its own sake. (The time commitment for Math 22 is standard.)

Regardless of which calculus course you take, keep in mind that it is important to absorb ideas thoroughly. It's a bad idea to push yourself too far too fast.

For more guidance on choosing your first math course at Harvard please read the pamphlet *Beyond Math 1: Which math course is for you?* which you can obtain from the undergraduate section of the Math Department website (that site link is here) .

2. How to structure a good program

No single program is ideal for all math concentrators. You should design your curriculum based on your background, interests, and future plans. You are strongly urged to consult with your academic advisor or the Director of Undergraduate Studies in deciding which courses are best suited for you. Do not plan to meet with your advisor on the last day for Crimson Cart submission because advisors usually don't have more than a few minutes to spend with each student that day. Make an appointment with your advisor well before the Crimson Cart submission date. You should allot about half an hour, so you can discuss your plan of study in depth.

LEARNING TO WRITE PROOFS: Math 22, 25, 101, 112, and 121 are five courses where you can learn to write and analyze proofs and rigorous mathematical arguments. It is best not to take an upper-level math courses without a reasonable comfort level with the language of rigorous mathematics. These courses will give you the requisite comfort level.

• Math 101 serves three main goals. It lets a student sample the three major areas of mathematics: analysis, algebra, and topology/geometry; it introduces the notions of rigor and proof; and it lets the student have some fun doing mathematics. If you are considering concentrating in Mathematics but are not sure that you are up for Math 22, 25 or 55, or if you simply want a glimpse of what "higher" math is all about, you are urged to include Math 101 early in your curriculum. Math 101 can be taken concurrently with Math 21a or 21b or Math 22a or 22b. If you have had some experience with rigorous proofs and want a different taste of "higher" math, you might consider Math 141 or Math 152 in the fall. Neither Math 101 nor Math 152 is appropriate for people from Math 25, Math 55. People who took Math 22 can freely take Math 101 and 152. (Note that Math 101 is offered in both the fall and spring semesters)

• Math 22, 25 and 55 are the three introductory courses for people with strong math interests coming into Harvard. Math 25 and 55 are much more intensive than Math 22, but require much more out of class time. People who don't wish to make the time commitment will do well to choose Math 22. Meanwhile Math 55 should be taken only by students with extensive college level math backgrounds. Each year one or two first-year students ask to skip the Math 25/55 level and start with Math 122 or another 100-level course. The Department, based on many years of experience, strongly discourages this. To elaborate: Even if you have taken several years of math at another university, even if you have 'seen' every topic to be covered in Math 25 or 55, you will not be bored in these courses. The topics covered in Math 25 and 55 are not as important as the level and the depth of mathematical sophistication at which they are taught. Taking Math 25 or 55 is the most intense mathematical experience you are going to have in any Harvard course, shared with the most talented of your peers. You may learn more advanced material in other 100- and 200-level courses, but never with the same breadth and depth as in Math 25 or 55. These courses are not taught in any other university because no other university has the same caliber of first-year mathematicians. And the courses are simply a lot of fun. Many students who have skipped 25 and 55 have been dissatisfied with their decision. In any event, you must speak with the Director of Undergraduate Studies if you plan to skip the Math 21-55 level.

• Math 112 and Math 121 are courses suitable for students from Math 21, and they provide an alternative entry-point for the department's more advanced courses in analysis and algebra respectively. Math 112 should not be normally be taken by students who have been through 25 or 55; and Math 121 should not be taken by students who have had one of the courses Math 22a, 25a or 55a. (Math 22a,b people can take Math 112, and Math 21a people who take Math 22b can take

Math 121). If you are a second year student and have taken Math 21 but are not yet comfortable with writing proofs, then consider including these courses in your plan of study.

If you have taken Math 22, 25 or 55, or if you have taken Math 21 and gained some experience in writing proofs through courses such as Math 101, 112 and 121, then you are ready to take some of the courses at the 100-level that form the core of the Mathematics curriculum. Most of the courses at this level can be classified as belonging to one of the three main streams of mathematics: *analysis*, *algebra*, and *geometry and topology*. Courses belonging to these areas are numbered in the ranges 110–119, 120–129 and 130–139 respectively.

CENTRAL COURSES: In each of these three streams, there are two courses which are somewhat more central in the sense that their material is used in ubiquitously in mathematics. These central courses are:

- Math 113. Analysis I: Complex Function Theory
  Math 114. Analysis II: Measure, Integration and Banach Spaces

- Math 122. Algebra I: Theory of Groups and Vector Spaces
  Math 123. Algebra II: Theory of Rings and Fields

- Math 131. Topology I: Topological Spaces and the Fundamental Group
  Math 132. Topology II: Smooth manifolds

It is not necessary to include all six of these courses in your plan of study, but here are some points to bear in mind

• Students from Math 55 will have covered most of the material in Math 122 and Math 113. If you have taken Math 55, you should look first at Math 114, Math 123 and the Math 131-132 sequence.

• With the exception just noted, you should consider including Math 122 early on in your curriculum. Algebra is a basic language of modern mathematics, and it is hard to comprehend advanced material without some familiarity with groups and related topics in algebra. The same remark applies to Math 123, to a lesser degree.

• By the same token, Math 113 should also be taken early on as complex analysis is used in many other fields of mathematics. You will also find the topology you learn in Math 131 useful in many other areas: amongst other things, it provides the mathematical language with which to discuss continuity and limits in wide generality.

• Math 123 cannot be taken before Math 122; but in the other two streams, the courses can be taken in either order. Thus, Math 114 can be taken before or after Math 113, and Math 132 before 131.

• You should try to fulfill the distribution requirement (i.e., the requirement to take at least one course in analysis, algebra, and geometry/topology) early in your academic career. By your third or fourth year, you should be exposed to the main branches of mathematics; then you can choose the department's advanced courses. In any case, most 200-level courses assume (at least informally) familiarity with the basic tools of analysis, algebra, and topology.

OTHER COURSES AT THE 100 LEVEL: At this level, there are many other courses to choose, some being:

• Dyamical systems in Math 118; convexity and optimization in Math 116, number theory in Math 124 or Math 129; classical geometry in Math 130; differential geometry in Math 136; algebraic geometry in Math 137, probability in Math 154, logic and set theory in Math 141 and Math 145; combinatorics in Math 155. Courses that are not listed here are not given every year; check the Harvard course catalogue for these.

• Consider taking a tutorial (Math 99r) during the second or third year, or the fourth year. (Even so, tutorials are not required by the concentration.) Many students find the tutorial to be one of the best courses they took at Harvard. Tutorials satisfy the Mathematics Expository Writing requirement and often lead to senior thesis topics. More about tutorials appears below.

• Students wishing to take a rigorous course in mathematical logic in years when Math 141 or 145 are not offered at Harvard should consider taking logic courses at M.I.T. In any event, the Harvard courses offer a good introduction to model theory, set theory and recursion theory — the three main branches of mathematical logic. Students interested in the more philosophical aspects of logic and/or in proof or set theory may want to take Philosophy 140 or Philosphy 144; and those interested in mathematics of computation should look into Computer Science 121 and some of the other theoretical Computer Science courses.

• Students interested in Combinatorics should look at Math 155, and may also want to look up M.I.T.'s listings in that area. (If you want M.I.T. courses to count for the concentration credit, you must get permission in advance from the Director of Undergraduate Studies.)

• Students are encouraged to take courses from a variety of professors in the department and not just to "follow" one teacher. It is advisable to be exposed to different views and styles of doing mathematics.

200-LEVEL COURSES: 100, 200 – WHAT'S THE DIFFERENCE? The difference between 100-level and 200-level courses is fairly easy to summarize: 100-level courses are designed for undergraduates, whereas the 200-level courses are generally designed for graduate students. As far as course material goes, the 100-level courses are designed to offer a comprehensive view of all the major fields in pure mathematics. They emphasize the classical examples and problems that started each field and they lead to some of the fundamental results that motivated the further development of the field. In contrast, a 200-level course will assume you understand the basic ideas in the field. A 200-level course will set out the systematic, abstract foundations for a field and develop tools needed to get to the present frontiers.

The 100-level courses give you a good overview of mathematics, they foster intellectual growth, and they prepare you for your chosen career. This is not true of 200-level courses. These courses assume that you are interested in the subject, and that you are already fairly certain of becoming an academic mathematician. The amount you learn in such a course is often also entirely up to you. Your prerequisites, though correct according to the course catalog, may be entirely inadequate.
    Many courses are paired into 100-level and 200-level sequences:

Corresponding 100-level, 200-level Courses

Math 114 $\rightarrow$ Math 212 (Real Analysis)
Math 113 $\rightarrow$ Math 213a,b (Complex Analysis and Riemann surfaces)
Math 122/123 $\rightarrow$ Math 221 (Commutative algebra)
Math 129 $\rightarrow$ Math 223a,b/229 (Algebraic Number Theory/Analytic Number Theory),
Math 131 $\rightarrow$ Math 231a,b (Algebraic Topology)
Math 132/136 $\rightarrow$ Math 230a,b (Differential Geometry)
Math 137 $\rightarrow$ Math 232a,b (Algebraic Geometry)

Other 200-level courses are harder to classify, but cover topics equally central to modern mathematics. For example, Math 222 and Math 224 are courses on Lie groups and Lie algebras that draws on background material from analysis, algebra and geometry.

SKIPPING 100-LEVEL PRECURSORS? Students are strongly discouraged from taking any 200-level course before taking its 100-level precursors. Although it is possible in principle to learn a general abstract topic on the basis of the logic of its definitions and theorems alone, it is almost impossible to appreciate their significance and "feel" without studying the more down-to-earth background which led to them. Moreover, students are well advised to take basic classes in algebra, topology, and analysis before exploring the graduate curriculum: often a basic familiarity with other areas will be an assumed prerequisite. Certainly, it can't hurt. However, even this may not suffice.

Some graduate courses (notably 212, 213a, 221, 231a) often conform better to undergraduate expectations (set material, careful pace, motivation); the best way to tell whether this is going to happen is to go to the class yourself and find out. Beware, though: Often these courses start in a user-friendly way (presenting simple definitions, for example), then speed up tremendously as time goes on.

WHY TAKE 200-LEVEL COURSES? The reasons for not taking 200-level courses are numerous. However, there are some equally good reasons for taking them. You will be treated like a graduate student, which is good if you want to be treated like one. There isn't much review of topics you may have already covered, requirements are fairly minimal, and, most importantly, you can learn a lot of substantial mathematics. (If this is what you want, tutorials are another good option. While they are undergraduate courses, one generally learns graduate material in them.)

A student who is considering graduate school in mathematics may want to include at least one 200-level course in their program (and, likewise, write a senior thesis) to get a taste of the likes of graduate school.

3. Other types of courses

TUTORIALS: Tutorials are not required, but many students take a tutorial during their second or third year. Typically one or maybe two are offered every semester.

Tutorials (Math 99r) is less formal and structured than regular courses, but it require much more involvement on the part of the student – students in the tutorials make presentations to the rest of the class and write papers on the topics at hand. Very frequently a topic studied in a tutorial leads naturally to a senior thesis. And a paper written for the tutorial generally satisfies the Math concentration's expository writing requirement.

The department places a description of the fall tutorial(s) on the Math 99r Canvas website at some point before the spring preregistration weeks; and a description of the spring tutorial is on that website at some point before the fall preregistration weeks. There is also a tutorial preview meeting during the course registration period.

Ordinarily only one Math 99r can count towards the concentration requirements.

All questions regarding tutorials may be addressed to the Director of Undergraduate Studies or the Undergraduate Program Coordinator, Cindy Jimenez (cindy@math.harvard.edu).

READING COURSES (60R AND 91R): Honors candidates in their last year at Harvard can choose to enroll in Math 60r to allow more time for senior thesis work. A person can take Math 60r in the fall

and/or spring semester. Math 60r is SAT/UNS only and does not count for concentration requirements. A person taking Math 60r in the fall must submit a one or two page plan of thesis (including at least a preliminary bibliography) to Cindy Jimenez (cindy@math.harvard.edu) by 4 pm of the last day of the fall reading period in order to get the SAT grade.

If you want to learn a particular topic not covered in a regular course or a tutorial, you can consider taking a Math 91r reading course with a faculty member. To do this, you must find a faculty member willing to supervise your reading, as well as secure approval from the Director of Undergraduate Studies. Make sure that you, your supervisor, and the Director of Undergraduate Studies clearly agree on the topic, structure, frequency of meetings, and the grade requirements before you sign up for Math 91r. You should know exactly what is expected of you and how much guidance to anticipate. Ordinarily, Math 91r will not count for concentration requirements.

CROSS-REGISTRATION AT M.I.T.: Students can cross-register to take a course at M.I.T. This may be a useful option in years when a particular course is not offered at Harvard. Logic and Combinatorics offerings at M.I.T. have proven especially popular with Harvard students. Generally, classes at M.I.T. start on a different date then Harvard courses so be aware of this when you register for an MIT course. You can get concentration credit for M.I.T. courses if you talk to the Director of Undergraduate Studies *before* registering. Information about how to cross register can be found in the Harvard Handbook for Students.

By the way: If you are taking an M.I.T. course, you don't have to walk all the way down Mass. Ave. or even pay for the bus to get to class: you can use the Harvard Medical Area (M2) shuttle bus, which runs from Quincy Square (in front of Lamont) straight to M.I.T.

RELATED FIELDS: Keep in mind that the concentration requirements for Mathematics require twelve half- courses, but only eight of those need to be listed under "Mathematics" in the Course Catalog. You are encouraged to round out your studies by including courses listed as "Related Fields" in the mathematics section of the Handbook for Students.

4. Sample Programs

The programs listed below should not be followed literally; they may not be balanced in workload between the fall and the spring semesters; nor are all the courses listed necessarily offered every year. They are presented only to serve as examples of possible programs. You should determine *your* unique program in consultation with your math faculty advisor or the Director of Undergraduate Studies.

If you start in Math Ma,b your first year, you can continue with Math 1b in the fall of your second year, then Math 21a in the spring of that year. Your third year fall could be Math 21b and Math 101 taken concurrently. Your third year spring might be Math 121 and Math 112, and then Math 130 in

the spring of your fourth year could be your eighth math course. Sprinkled along the way are four more related field courses. What follows could be one way to make this work with some Computer Science (CS) courses:

Year 1	Year 2	Year 3	Year 4
Math Ma	Math 1b CS 50	Math 21b Math 101	Math 110 CS 121
Math Mb CS 1	Math 21a CS 51	Math 112 Math 121	Math 130 CS 124

If you start with Math 1a,b in your first year, you can continue with Math 21a,b in the second year. Students who start with Math 1b in the fall of their first year normally take Math 21a in the spring; some choose also to take Math 21b concurrently with 21a (but you certainly don't have to). Some students who start with Math 1a,b sequence freshman year choose to take Math 22 or 25 their sophomore year instead of Math 21 to get a first feel for proofs and abstraction; but most people get that by taking Math 101, 112 or 121. Here is a possible program:

Year 1	Year 2	Year 3	Year 4
Math 1a	Math 21a	Math 112	Math 113
Math 1b	Math 21b	Math 121	Math 131
CS 50	CompSci 51	Phys 15a	Math 122
	Math 101	Stat 110	Phil 144

Students who start with 21a,b in their first year can take 101 either concurrently with one of the Math 21's or in their second year along with 112, 122 and/or 131. Many students also take Physics 15a,b,c or Computer Science 51 to see how mathematics applies to other disciplines. Here is a possible program:

Year 1	Year 2	Year 3	Year 4
Math 21a	Math 122	Math 141	Math 231a
Phys 15a	Math 131	Math 124	Math 114
Math 21b	CompSci 51	Math 99r	Math 231b
Math 101	Math 112	Math 132	EC 2052

A student entering with a strong interest in mathematics would most likely start with Math 22, 25 or 55 during the first year. (Math 55 if a strong background also.) A sample program might look like this:

Year 1	Year 2	Year 3	Year 4
Math 22a or 25a	Math 122	Math 114	Math 60r
Physics 15a	Math 131	Math 99r	Math 212
Math 22b or 25b	Math 123	Math 129	Math 222
Physics 15b	Math 113	Math 132	Math 137

Consider a person with a strong interest in mathematical physics who wants to concentrate in Mathematics with Physics as an allied concentration. If this person has the Math 1a,b equivalent on entering, then starting with Math 22a,b or Math 25a,b (or Math 55a,b) with the Physics 16 (or 15a), 15b and 15c sequence makes sense. Looking ahead, 100-level math courses of particular use to physicists are Math 113, 132 and 136. The following is a program for this student that will fulfill the Math concentration part of the joint concentration requirements. (The student should talk with the Physics Director of Undergraduate studies to plan the physics portion.)

Year 1	Year 2	Year 2	Year 4
Math 22a or 25a	Math 131	Math 113	Math 230a
Physics 16	Math122	Math 132 or 136	Math 99r
Math 22b or 25b or 55b	Phys 15c	Physics 143b	Math 230b
Physics 15b	Phys 143a	Phys 181	Math 123

A primary concentration in Mathematics with Computer Science as an allied concentration is common. Mathematics courses of particular value here would be Math 141 (introduction to mathematical logic), Math 124 (number theory including primality tests and applications to codes), Math 130 (on axiomatic foundations of geometry), Math 155 (combinatorics), Math 152 (discrete mathematics) and Math 154 (probability).   A possible program which fulfills the Math concentration requirements is given below. Consult with the Computer Science Head Tutor to plan that portion of the course work.

Year 1	Year 2	Year 3	Year 4
Math 21a	Math 121	Math 122	Math 141 or Math 154
Math 101	CS 121	CS 207 or 226r	Math 191
Math 21b	Math 112	Math 155	Math 152
CS 51	CS 124	Math 130	Math 124 or AM 111