

**Beyond Math 1:  
Which math course is for you?  
(2020-2021)**

If you have completed the Math 1a/1b sequence at Harvard or if you have had the equivalent material elsewhere, you may be wondering which course is for you. The mathematics department provides a variety of options which you should consider based on your academic interests and your background. With exceedingly rare exceptions, students in your position are advised to take one of Math 18a, 19a, 18b/19b, 21a, 21b, 22a, 23a, 25a, 55a. You can also take Math 101 concurrently with any of Math 18-22. (The School of Engineering and Applied Sciences also offers Applied Math 22a,b which covers much the same topics from Math 21a,b but in different order.) This pamphlet describes the Mathematics Department's offerings and should help you decide which course is for you.

- **Math 18a:** This course is taught in the fall only. Math 18a covers the concepts and techniques of multivariable calculus most useful to those studying the social sciences, particularly economics. The course also serves as an introduction to mathematical modeling in the social sciences and economics. Math 18a should not be taken in addition to Math 21a, but Math 19b or Math 21b may be taken before or after Math 18a.
  
- **Math 19a:** Math 19a is given in just the fall; it teaches multivariable calculus and differential equations for applications to the life sciences. It is recommended by those taking the new Life Science 1a,b courses and by the life science concentrations (this means Biological Anthropology, Chemical and Physical Biology, Human Evolutionary Biology, Molecular and Cellular Biology, Neurobiology, Organismic and Evolutionary Biology, and Social and Cognitive Neuroscience.) In particular, the focus in Math 19a is on differential equations, both linear and non-linear in one or more variables. Related techniques are also presented. Math 19a has a second focus which is mathematical modeling for life science problems. This course is preferable to Math 21a for those majoring in a life science except for students who plan to take Physics 15/16.
  
- **Math 18b/19b:** This one course is given only in the spring semester. Math 18b/19b is a follow-on course for both Math 18a and Math 91a. It is for people interested in life sciences or social sciences or economics. The course teaches linear algebra with enough probability and statistics to forgo the beginning statistics courses such as Stat 100, 102, 104. Some programming will also be taught. All of the linear algebra in Math 21b is taught; The differential equations in Math 21b is traded for probability and statistics.
  - \* NOTE: Math 21b can be taken after Math 19a and Math 18b/19b can be taken after Math 21a. (But Math 19a and Math 21a can't both be taken. Likewise, Math 18b/19b and Math 21b both be taken.)
  
- **Math 21a:** This course covers the basics of multivariable calculus in two and three dimensions: Curves and surfaces, functions and their derivatives, the calculus of variations, multi-variable integration, integration on curves and surfaces, multivariable generalizations of the fundamental theorem of calculus. In short: Math 21a teach the tools and intuition for dealing with basic multivariable problems. Math 21a is given in both the fall and spring semester.

- Math 21b: This course covers the basics of linear algebra in dimensions 2, 3 and higher. A significant part of the course uses the linear algebra to study ordinary and partial differential equations. Math 21a and Math 21b can (in principle) be taken in either order, but most students take Math 21a first. (Also, the abstract parts of Math 21b are probably easier to follow having taken Math 21a.)

- Math 22a,b: This course covers multivariable calculus and linear algebra for students interested in theoretical sciences. It covers the same topics as Math 21a,b but with more rigor. Students are taught techniques of proof and mathematical reasoning. The workload and content is comparable with the 21 sequence. But unlike in the later, the linear algebra and calculus is more interlinked. Math 22b can be taken after Math 21a and Math 21b can be taken after Math 22a

- Math 23a,b,c: This sequence likewise treats multivariable calculus and linear algebra in a rigorous, proof-oriented way. It differs from Math 21a,b and Math 22a,b by virtue of it being taught in a different format (one section has students watching videos before class and then doing presentations and problem solving in class). Math 23a,b,c also makes extensive use of computational tools (R-script). Various applications for other science and social science concentrations are presented also. Math 23a is the fall course. Students can take either Math 23b or Math 23c in the spring (Math 23c is geared somewhat towards data science applications.) Note that this course is typically more time consuming than Math 21a,b (but for the most part, not as time consuming as Math 25a,b).

- Math 25 and 55: These are theory courses that should be elected only by those students who have a particular interest in and enjoyment of abstract mathematics, as well as a solid understanding of one-variable calculus. (But note that many of the students in these courses go on to concentrate in other sciences.) These courses assume a willingness to think rigorously and abstractly about mathematics; and they require a willingness to work hard. Both courses study multivariable calculus and linear algebra plus many very deep, related topics. These courses come with an iron clad guarantee that you will be challenged by the mathematics.

\* CHOOSING BETWEEN MATH 22, 23, 25 OR 55: Math 25 differs from Math 22 and Mathematics 23 by virtue of the work load in Mathematics 25 being significantly more than in than in these other courses; but then Mathematics 25 covers more material. Even so, any given course that asks for Math 25 as a prerequisite accepts Math 23 and Math 22 as well. As noted above, Math 23 differs from Math 22 in that material in Math 23 is often learned from video lectures whereas classroom time is dedicated to presentations and problem solving. Math 23 also makes extensive use of computational tools. Meanwhile, Mathematics 55 differs from Mathematics 25 in that the former assumes an extremely strong proof oriented mathematics experience. Entrance into Mathematics 55 requires the consent of the instructor.

\* SKIPPING MATH 25/55: Every year a few first year people with advanced backgrounds want to skip Math 25 and Math 55 and start with a 100- or 200- level course. Based on many years of experience, we discourage this. You may learn more advanced material in higher level courses, but never at the same speed and intensity as in Math 25 or 55. Moreover, you are learning more

than just a body of mathematics in these courses. You are also learning how to 'be' a research mathematician (as opposed to one who only does well in math courses). If, in spite of this warning, you think that taking a higher level course as a freshman would best serve your needs, you should speak to the Director of Undergraduate Studies in Mathematics.

- Applied Math 22: The Applied Math 22a,b sequence is like Math 21a,b with regards to content (although the material is presented in a different order). Math 22a,b is also taught from a somewhat more applied point of view. Both Math 22a and Math 22b are taught in a single large lecture hall format, whereas Math 21a,b are taught in small sections that are designed to maximize student/ teacher interactions. (Mathematics concentrators can use Applied Math 22a,b as as related field courses for concentration credit if they do not also take Math 21a,b.)

#### OTHER COURSES:

- Math 101: Math 101 (fall semesters) is designed to give people with a Math 1b level background and with interest in mathematics a taste of what modern mathematics is all about. This course can be taken concurrently with Math 21a or 21b or 22a. It is not to be taken with Math 23, 25 or 55 (without special permission). Math 101 also gives a good background for writing and following mathematical proofs. This skill will be needed in any event for most higher level math courses. This skill is also taught in Math 22, 23 and Math 25, and in a few other 100 level courses.