DEPARTMENT OF MATHEMATICS
HARVARD UNIVERSITY

ADVANCES IN MATHEMATICAL PHYSICS: A CONFERENCE IN HONOR OF ELLIOTT H. LIEB ON HIS 90TH BIRTHDAY

July 30 - August 1, 2022
July 30 - 31: Harvard University Science Center, Lecture Hall B
August 1: Harvard University Science Center, Lecture Hall C

Conference Schedule

Saturday, July 30, Hall B	Sunday, July 31, Hall B	Monday, August 1, Hall C
8:45 a.m. - 9 a.m. Refreshments	**8:45 a.m. - 9 a.m.** Refreshments	**8:45 a.m. - 9 a.m.** Refreshments
9 a.m. - 9:45 a.m. Jan Philip Solovej	**9 a.m. - 10 a.m.** Hugo Duminil-Copin	**9 a.m. - 9:45 a.m.** Yoshiko Ogata
9:45 a.m. - 10:30 a.m. László Erdös	**10 a.m. - 10:30 a.m.** Tea Break	**9:45 a.m. - 10:30 a.m.** Hal Tasaki
10:30 a.m. - 11 a.m. Tea Break	**10:30 a.m. - 11:15 a.m.** Jürg Fröhlich	**10:30 a.m. - 11 a.m.** Tea Break
11 a.m. - 11:45 a.m. Robert Seiringer	**11:15 a.m. - 12 p.m.** Bertrand Halperin	**11 a.m. - 11:45 a.m.** Bruno Nachtergaele
11:45 a.m. - 12:30 p.m. Rupert Frank	**12:00 p.m. - 1:30 p.m.** Lunch	**11:45 a.m. - 12:30 p.m.** Alessandro Guiliani
12:30 p.m. - 2 p.m. Lunch	**1:30 p.m. - 2:15 p.m.** Jun Yin	**12:30 p.m. - 2 p.m.** Lunch
2 p.m. - 2:45 p.m. Simone Warzel	**2:15 p.m. - 3 p.m.** Sabine Jansen	**2 p.m. - 2:45 p.m.** Ron Peled
2:45 p.m. - 3:30 p.m. Benjamin Schlein	**3 p.m. - 3:30 p.m.** Tea Break	**2:45 p.m. - 3:30 p.m.** Mathieu Lewin
3:30 p.m. - 4 p.m. Tea Break	**3:30 p.m. - 4:45 p.m.** A Review of Lieb's Work	**3:30 p.m. - 4 p.m.** Tea Break
4 p.m. - 4:45 p.m. Rafael Benguria		**4 p.m. - 4:45 p.m.** Eric Carlen

ADVANCES IN MATHEMATICAL PHYSICS: A CONFERENCE IN HONOR OF ELLIOTT H. LIEB ON HIS 90TH BIRTHDAY

Conference Program
Saturday, July 30, Hall B

9 a.m. - 9:45 a.m.
Jan Philip Solovej
Minimal Output Entropy for Invariant Quantum Channels

Abstract: I will introduce quantum channels, i.e., completely positive trace preserving maps, that are invariant under group actions. For the groups SU(N) and SU(1,1) I will introduce explicit invariant quantum channels. For certain representations of SU(N) I will determine the minimal output entropy of these, while for SU(1,1) there are essentially only conjectures. I will briefly mention the relation to the Wehrl entropy and discuss what is known for the minimal Wehrl entropy. This is joint work with Elliott Lieb.

9:45 a.m. - 10:30 a.m.
László Erdös
CLT and Extremal Statistics for Non-Hermitian Random Matrices

Abstract: We establish a general central limit theorem for the linear statistics of the eigenvalues and a precise three-term asymptotic expansion, with an optimal estimate of the error term, for the rightmost eigenvalue of an n by n random matrix with independent identically distributed entries as n tends to infinity. All terms in the expansion are universal. We explain the major difficulties of the spectral analysis for random non-Hermitian matrices compared with the more conventional Hermitian situation.

11 a.m. - 11:45 a.m.
Robert Seiringer
Ground state energy and effective mass of a strongly coupled polaron

Abstract: We explain recent bounds on the quantum corrections to the (classical) Pekar approximation of the ground state energy of the Fröhlich polaron model in the strong coupling limit, and their consequence on the effective mass.

11:45 a.m. - 12:30 p.m.
Rupert Frank
Minimal magnetic fields supporting a zero mode

Abstract: We present some recent results on minimal magnetic fields that admit a zero mode for the associated Pauli equation. The importance of zero modes to the problem of stability of matter in the presence of magnetic fields was observed by Frohlich, Lieb and Loss and their existence was shown by Loss and Yau. We show that the Loss-Yau modes arise as optimizers of a variational problem where the minimality is measured in terms of the 3-norm of the vector potential. We also prove existence of optimal modes when it is measured in terms of the 3/2-norm of the field. Both problems have a conformal invariance and we discuss similarities and dissimilarities with Lieb's solution of the Hardy-Littlewood-Sobolev problem.

The talk is based on joint works with Michael Loss.

2 p.m. - 2:45 p.m.
Simone Warzel
Semiclassical analysis of mean-field quantum spin systems

Abstract: In 1973 Elliott Lieb showed that the statistical mechanics of quantum spin systems becomes classical in the limit of large spin quantum number. The analysis is based on Bloch coherent states and what is nowadays known as Berezin-Lieb inequalities. This talk will start from an application of these techniques to mean-field models, for which one may derive semiclassical expressions for the specific free energy. Many prominent mean-field models like the quantum Curie-Weiss Hamiltonian are known or conjectured to exhibit phase transitions which at zero temperature separate gapped ground-state phases. In the main part of the talk, I will discuss a simple semiclassical description of the spectral gap in such models.

2:45 p.m. - 3:30 p.m.
Benjamin Schlein
Ground state energy of a gas of hard spheres in the Gross-Pitaevskii regime

Abstract: We consider a gas of N bosons moving on the three-dimensional unit torus and interacting through a hard sphere potential with radius of order 1/N. We prove an upper bound for the ground state energy, valid up to errors that vanish in the limit of large N. This is joint work with G. Basti, S. Cenatiempo, A. Olgiati and G. Pasqualetti.

4 p.m. - 4:45 p.m.
Rafael Benguria
New bounds on the excess charge for atomic systems

Abstract: In this talk, using a technique introduced by P.T. Nam in 2012 and the Coulomb Uncertainty Principle, I will present the proof of new bounds on the excess charge for non relativistic atomic systems, independent of the particle statistics. These new bounds are the best bounds to date for bosonic systems for all values of the atomic number Z and they are also the best bounds for fermionic systems with $Z \leq 26$ (i.e., up to the chemical element iron). This is joint work with J. M. Gonzalez-Brantes and T. Tubino.

Sunday, July 31, Hall B

9 a.m. - 10 a.m.
Hugo Duminil-Copin
Emergent symmetries in critical 2D statistical physics

Abstract: A great achievement of physics in the second half of the twentieth century has been the prediction of conformal symmetry of the scaling limit of critical statistical physics systems. Around the turn of the millennium, the mathematical understanding of this fact progressed tremendously in two dimensions with the introduction of the Schramm-Loewner Evolution and the proofs of conformal invariance of the Ising model and dimers. Nevertheless, as for today the understanding remains restricted to very specific models. In this talk, we will present a (at the time non-rigorous) strategy towards proving conformal invariance of a large class of models based on the links between percolation models and the six-vertex model. We will also present progress in some of the important steps in this strategy. An important role will be played by a geometric interpretation of integrability, and the notion of universality.

10:30 a.m. - 11:15 a.m.
Jürg Fröhlich
The Classical Periphery of Quantum Mechanics

Abstract: In this talk I consider regimes of Quantum Mechanics that can be described in classical terms. Such regimes constitute what I call the "Classical Periphery/Skin of Quantum Mechanics." I won't develop the general theory, but illustrate in terms of some examples. Most probably I will present a study of tracks traced out by quantum- mechanical particles propagating in detectors. These tracks turn out to be close to classical particle trajectories.

I will begin my talk with some general comments on the notion of "events" in Quantum Mechanics and their role in understanding "state reduction" (ETH-Approach to QM), as manifested in measurements and observations.

11:15 a.m. - 12 p.m.
Bertrand Halperin
Fractional Statistics, Edge Modes, and Interferometry in Fractional Quantized Hall States

Abstract: A quantized Hall phase of a two-dimensional electron system is a state which, in the absence of disorder, has an energy gap in the bulk, but has gapless modes at the sample boundaries, which form a chiral one-dimensional metal. A sample with several narrow constrictions that allow tunneling of quasiparticles from one edge to another can give rise to interference oscillations in the electrical resistance as a function of applied magnetic field or gate voltages. In fractional quantum Hall states, there are quasiparticles that have fractional electron charge and exhibit fractional or non-abelian statistics, and these can have dramatic effects on the interference patterns. Though observations and interpretations can be complicated by effects of disorder and long-range Coulomb interactions, the signature of fractional statistics has been directly observed in recent experiments.

1:30 p.m. - 2:15 p.m.
Jun Yin
Delocalization of random band matrices in high dimensions, spontaneous renormalization

Abstract: One famous conjecture in quantum chaos and random matrix theory is the so-called phase transition conjecture of random band matrices. It predicts that the eigenvectors' localization-delocalization transition occurs at some critical bandwidth $W_c(d)$, which depends on the dimension d. The well-known Anderson model and Anderson conjecture have a similar phenomenon. It is widely believed that $W_c(d)$ matches $1/\lambda_c(d)$ in the Anderson conjecture, where $\lambda_c(d)$ is the critical coupling constant. Furthermore, this random matrix eigenvector phase transition coincides with the local eigenvalue statistics phase transition, which matches the Bohigas-Giannoni-Schmit conjecture in quantum chaos theory. We proved the eigenvector's delocalization property for most of the general d>=7 random band matrix as long as the size of this random matrix does not grow faster than its bandwidth polynomially. In other words, delocalization occurs as long as bandwidth W is larger than L^ε, for matrix size L, and some $\varepsilon>0$. It is joint work with H.T. Yau (Harvard) and F. Yang (Upenn).

2:15 p.m. - 3 p.m.
Sabine Jansen
Free energies and effective interfaces for the high-density Widom-Rowlinson model

Abstract: The Widom-Rowlinson model is one of the few continuum models of classical statistical mechanics for which the existence of a phase transition has been rigorously proven. In the single-color version the energy is roughly the area covered by a union of balls. We investigate the model in a finite 2-dimensional box in the limit of large density and ask (1) what is the probability that the union of disks centered at particles is approximately equal to a given shape, say a disk, and (2) how should we think of interface fluctuations in the vicinity of the disk. The questions are motivated by the conjectured metastable behavior for a dynamic model in which particles are born and die. The talk presents partial answers as well as connections with geometric inequalities and the parabolic hull and growth processes introduced for studying the convex hull of a random set of points. Based on joint work with Frank den Hollander, Roman Kotecký, and Elena Pulvirenti.

3:30 p.m. - 4:45 p.m.
A Review of Lieb's Work

Monday, August 1, Hall C

9 a.m. - 9:45 a.m.
Yoshiko Ogata
Classification of gapped ground state phases in quantum spin systems

Abstract: Recently, classification problems of gapped ground state phases have attracted a lot of attention in quantum statistical mechanics. We explain about operator algebraic approach to these problems.

9:45 a.m. - 10:30 a.m.
Hal Tasaki
Variations on a Theme by Lieb, Schultz, and Mattis: Unique Gapped Ground States, Symmetry-Protected Topological Phases, Edge States, Spin Pumping, and all That in Quantum Spin Chains

Abstract: I will show how one can make use of the twist operator introduced by Lieb, Schultz, and Mattis in 1961 and refined by Affleck and Lieb in 1986 to define, in an elementary manner, topological indices for U(1)-invariant quantum spin chains. We shall use the indices to prove interesting properties of quantum spin chains, including the generalized Lieb-Schultz-Mattis theorem, the existence of a symmetry-protected topological phase transition between the AKLT model and a trivial model, the existence of edge modes in the Haldane phase, and the existence of nonzero spin pumping in a process that connects the AKLT and the trivial model continuously.

11 a.m. - 11:45 a.m.
Bruno Nachtergaele
Lieb-Robinson Bounds: Seed of a Revolution

Abstract: I discuss the pivotal role of Lieb-Robinson bounds (Lieb and Robinson, 1972) in characterizing and utilizing locality properties of quantum lattice systems. In 2004, Hastings' generalization of the Lieb-Schultz-Mattis Theorem to multi-dimensional systems demonstrated how to make effective use of Lieb-Robinson bounds to study quantum lattice dynamics and states. A series of important new results in which Lieb-Robinson bounds featured in an essential way followed in quick succession. I conclude by reviewing some recent results on the classification and stability of gapped ground state phases and highlight the essential role of Lieb-Robinson bounds in their proofs.

11:45 a.m. - 12:30 p.m.
Alessandro Guiliani
Periodic striped states in Ising models with polynomial interactions

Abstract: I will discuss the problem of determining the ground states of 2D Ising models with nearest neighbor ferromagnetic and dipolar-like interactions and review known results, including the proof of existence of periodic striped minimizers in the presence of anti-ferromagnetic polynomial interactions decaying like $1/(\text{dist})^p$, with p larger than 4-ε. In the standard dipolar case, p=3, I will discuss the proof that the minimizers in the variational class of states whose domain walls are arbitrary collections of horizontal and/or vertical straight lines are periodic and striped. Based on joint works with Davide Fermi, Joel Lebowitz, Elliott Lieb, Robert Seiringer.

2 p.m. - 2:45 p.m.
Ron Peled
Random packings and liquid crystals

Abstract: Let T be a subset of R^d, such as a ball, a cube or a cylinder, and consider all possibilities for packing translates of T, perhaps with its rotations, in some bounded domain in R^d. How does a typical packing of this sort look like? One mathematical formalization of this question is to fix the density of the packing and sample uniformly among all possible packings with this density. Discrete versions of the question may be formulated on lattice graphs.

The question arises naturally in the sciences, where T may be thought of as a molecule and its packing is related to the spatial arrangement of molecules of a material under given conditions. In some cases, the material forms a liquid crystal - states of matter which are, in a sense, between liquids and crystals.

Elliott Lieb is a leader of this field. His work ranges from the highly influential 1970 paper with Ole Heilmann showing the absence of a phase transition in the monomer-dimer model, to the 2018 paper with Jauslin establishing a nematic liquid crystal phase in a monomer-dimer model with alignment-favoring interactions, finally settling a 1979 conjecture of Heilmann-Lieb.

I will review ideas from this topic, mentioning some of the predictions and the mathematical progress. Time permitting, I will elaborate on a recent result, joint with Daniel Hadas, on the structure of high-density packings of 2x2 squares with centers on the square lattice.

The talk is meant to be accessible to a general mathematical audience.

2:45 p.m. - 3:30 p.m.
Mathieu Lewin
Recent results in Density Functional Theory

Abstract: I will review Elliott's formulation of Density Functional Theory (DFT) from 1983 and present recent advances obtained in collaboration with him and Robert Seiringer.

4 p.m. - 4:45 p.m.
Eric Carlen
Elliott Lieb's work on operator and Trace inequalities and some recent developments

Abstract: Elliott Lieb's work in the first half of the 1970's on trace and operator inequalities, especially his work on the Wigner-Yanase-Dyson conjecture as well as related joint work with Araki and with Ruskai, has had a profound influence on many researchers in physics and mathematics. The lines of investigation that were opened with these papers continue to be actively developed even now, about 50 years later. We discuss this work from its origins through a selection of modern developments and new results.