(1) Rudin, Chapter 3, Problems 3, 6 (a)-(c), 7

(2) Suppose that \( \{a_n\} \) is a sequence of points in a metric space \((X, d)\), such that
\[
d(a_n, a_{n+1}) < c^2 d(a_n, a_{n-1}).
\]
for some \( c \in (0, 1) \). Show that \( \{a_n\} \) is a Cauchy sequence. (Fun Fact: This little problem is a key step in the proof of the implicit function theorem).

(3) Let \( \{a_n\} \) and \( \{b_n\} \) be two Cauchy sequences in \((X, d)\). Show that \( d(a_n, b_n) \) is a Cauchy sequence in \( \mathbb{R} \). (Fun Fact: This little problem is used in one method for constructing the real numbers).

(4) Suppose that \( \{a_n\} \) is a Cauchy sequence in \((X, d)\), and suppose that some subsequence \( \{a_{n_k}\} \) converges to a point \( p \in X \). Show that \( \{a_n\} \) converges to \( p \).