(1) Rudin, Chapter 1, Problems 6, 7.

(2) A real number $x \in \mathbb{R}$ is said to be *algebraic* if there exists integers $a_0, \ldots, a_n \in \mathbb{Z}$, not all zero, such that

$$a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n = 0.$$ 

Said differently, $x$ is a root of a polynomial with integer coefficients. Prove that there are only countably many algebraic numbers.

(3) Prove that the real numbers are uncountable. Conclude, in particular, that there are uncountably many numbers which are *not* algebraic.