• Start by printing your name in the above box.
• Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
• Do not detach pages from this exam packet or unstaple the packet.
• Please try to write neatly. Answers which are illegible for the grader can not be given credit.
• No notes, books, calculators, computers, or other electronic aids are allowed.
• Problems 1-3 do not require any justifications. For the rest of the problems you have to show your work. Even correct answers without derivation can not be given credit.
• You have 180 minutes time to complete your work.
<table>
<thead>
<tr>
<th>Problem 1) (20 points) No justifications are necessary</th>
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<td>20) T F</td>
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Problem 2) (10 points) No justifications are necessary.

a) (4 points) Match the objects with the definitions.

1-4  enter  vector field

\begin{align*}
\vec{F}(x, y, z) &= \langle x, y, z \rangle \\
\vec{F}(x, y, z) &= \langle -y, x, 0 \rangle \\
\vec{F}(x, y, z) &= \langle 0, z, 0 \rangle \\
\vec{F}(x, y, z) &= \langle -x, 0, -z \rangle
\end{align*}

b) (3 points) Match the surfaces with their names: (put O if no match)

1-4  enter  surface

\begin{align*}
x^2 + y^2 + 3z &= 0 \\
x^2 + y^2 - 3z^2 &= 1 \\
x^2 + y^2 + 3z^2 &= 1 \\
x^3 + 3y^2 &= 1
\end{align*}

c) (3 points) Match the space curves

1-4  parametrized curve

\begin{align*}
\vec{r}(t) &= \langle t, t^2, t^3 \rangle \\
\vec{r}(t) &= \langle \cos(3t), \sin(3t), t \rangle \\
\vec{r}(t) &= \langle (2 + \cos(t)) \cos(3t), (2 + \cos(t)) \sin(t), \sin(3t) \rangle \\
\vec{r}(t) &= \langle t, t \cos(3t), t \sin(3t) \rangle
\end{align*}
Problem 3) (10 points) No justifications are necessary

a) (5 points) We watch "angry birds" attacking on curves with acceleration \( r''(t) \). (The pictures show the \( xz- \) planes and the birds start with a constant velocity \( ⟨1, 0, 0⟩\).) Match the displayed curves \( r(t) \) with the formulas for accelerations.

\[
\begin{align*}
\vec{r}'(t) &= \langle 0, 0, \sin(t) \rangle \\
\vec{r}''(t) &= \langle 0, 0, -10 \rangle \\
\vec{r}''(t) &= \langle 0, 0, 10 \rangle \\
\vec{r}''(t) &= \langle 0, 0, -\sin(t) \rangle \\
\vec{r}''(t) &= \langle 0, 0, 0 \rangle 
\end{align*}
\]

b) (5 points) Match the formulas: (put O if no match)

<table>
<thead>
<tr>
<th>label</th>
<th>formula</th>
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<tbody>
<tr>
<td>A</td>
<td>( \vec{r}''(t) )</td>
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<tr>
<td>B</td>
<td>( \int_0^1</td>
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<tr>
<td>C</td>
<td>( \vec{r}'(t)/</td>
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<td>D</td>
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<tr>
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<tbody>
<tr>
<td>Curvature</td>
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<td>Unit tangent vector</td>
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<tr>
<td>Unit normal vector</td>
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<tr>
<td>Velocity</td>
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</tr>
<tr>
<td>Arc length</td>
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</table>
Problem 4) (10 points)

a) (5 points) Find a parametrization of the line \( L \) through the center of the two spheres \( x^2 + (y - 1)^2 + z^2 = 1 \), \((x - 5)^2 + y^2 + z^2 = 1\).

b) (5 points) Find the plane perpendicular to the line \( L \) for which the distances to the spheres are the same.

Problem 5) (10 points)

Johannes Kepler asked which cylinder or radius \( r \) and height \( 2h \) inscribed in the unit sphere has maximal volume. To solve his problem, use the Lagrange method and maximize the volume

\[
f = 2\pi r^2 h
\]

under the constraint that \( r^2 + h^2 = 1 \).

Problem 6) (10 points)

a) (6 points) Find the surface area of the surface

\[
r(u, v) = (v^2 \cos(u), v^2 \sin(u), v^2), 0 \leq u \leq \pi, 0 \leq v \leq 1.
\]

b) (4 points) Find the arc length of the boundary curve \( \vec{r}(u, 1) \) where \( 0 \leq u \leq \pi \).
Problem 7) (10 points)

Find the volume of the solid inside the cylinder

\[ x^2 + y^2 \leq 2 \]

sandwiched between the graphs of \( f(x, y) = x - y \) and \( g(x, y) = x^2 + y^2 + 4 \).

Problem 8) (10 points)

Find the flux of the curl of the vector field

\[ \vec{F}(x, y, z) = \langle x, y, z + \sin(\sin(y^2)) \rangle \]

through the torus

\[ \vec{r}(s, t) = \langle (2 + \cos(s)) \cos(t), (2 + \cos(s)) \sin(t), \sin(s) \rangle \]

with \( 0 \leq t \leq \pi \) and \( 0 \leq s < 2\pi \).

Problem 9) (10 points)

**Heron’s formula** for the area \( A \) of a triangle of side length \( x, y, 1 \) satisfies \( 16A^2 = f(x, y) \), where

\[ f(x, y) = -1 + 2x^2 - x^4 + 2y^2 + 2x^2y^2 - y^4 \]

Classify all the critical points of \( f \). Is there a global maximum of \( f \) and so for the area?

**Remark not to worry about:** The formula follows directly from Heron’s formula \( s = (a + b + 1)/2; A = \sqrt{s(s-a)(s-b)(s-1)}. \)
Problem 10) (10 points)

The anti derivative of the \textbf{sinc} function
\[
\frac{\sin(x)}{x}
\]
is called the \textbf{sine integral} \(\text{Si}(x)\). It can not be expressed in terms of known functions. Still we can compute the following double integral
\[
\int_0^\pi \int_x^\pi \frac{\sin(y)}{y} \, dy \, dx.
\]

Problem 11) (10 points)

Find the line integral of the vector field
\[
\vec{F}(x, y, z) = \langle -x^{10}, \sin(y), z^3 \rangle
\]
along the curve \(\vec{r}(t) = \langle \sin(t) \cos(5t), \sin(t) \sin(5t), t \rangle\) where \(0 \leq t \leq 2\pi\).

Problem 12) (10 points)

Find the area of the region enclosed by the curve
\[
\vec{r}(t) = \langle \cos(t), \sin(t) + \cos(2t)/2 \rangle,
\]
where \(0 \leq t < 2\pi\).
Problem 13) (10 points)

Find the flux of the vector field

\[ \vec{F}(x, y, z) = \langle \frac{x^3}{3}, \frac{y^3}{3}, \sin(xy^5) \rangle \]

through the boundary surface of the solid bound by the surface of revolution \( \vec{r}(t, z) = \langle (2 + \sin(z)) \cos(t), (2 + \sin(z)) \sin(t), z \rangle \) and the planes \( z = 0, z = 3 \). The surface is oriented so that the normal vector points outwards.