Math 118R Homework 1

Due Feb. 2nd, 2016

1. Analyze the following equations graphically: Sketch the vector field on the real line, find all the fixed points, classify their stability, and sketch the graph of $x(t)$ for some initial conditions.

\[
\begin{align*}
\dot{x} &= 4x^2 - 16 \\
\dot{x} &= x - x^3 \\
\dot{x} &= e^{-x} \sin(x)
\end{align*}
\]

2. In an electrical circuit with resistor $R$, capacitor $C$, and battery of constant DC voltage $V_0$, the charge, $Q(t)$, of the capacitor satisfies the differential equation

\[V_0 = R\dot{Q} + Q/C.\]

a) Analyze the equation graphically.

b) Find an explicit solution for every initial value $Q_0$.

3. The growth of cancerous tumors can be modeled by the Gompertz law

\[\dot{N} = -aN \ln(bN)\]

where $N(t)$ is proportional to the number of cells in the tumor, and $a, b > 0$ are parameters.

a) Analyze the equation graphically.

b) Use linear stability analysis to classify the fixed-point(s).

c) What is the meaning of the constants $a$ and $b$?

4. Show that the initial value problem $\dot{x} = x^{1/3}, x_0 = 0$, has an infinite number of solutions. (Hint: Construct a solution which is constant at first, then takes off.)