1. Evaluate \( \sum_{i}^{\infty} 1/n^4 \) by equating two different expressions for \( f'''(z) \), where \( f(z) = \pi \cot(\pi z) \).

2. Given a sequence of positive reals \( r_i \to \infty \), let \( N(r) = |\{i : r_i < r\}| \), and let \( \rho_1 = \inf \{ \alpha > 0 : \sum r_i^{-\alpha} < \infty \} \) be the critical exponent. Show that \( \rho_1 = \lim \sup_{r \to \infty} (\log N(r)/\log r) \).

3. Find all entire functions \( f(z) \) satisfying \( f(z+1) = 2f(z) \). Show there are infinitely many such functions of finite order with no zeros and with \( f(0) = 1 \).

4. Prove that \( \int_{0}^{1} \log \Gamma(t)dt = \log \sqrt{2\pi} \), using the duplication formula for \( \Gamma(2z) \).

5. For a fixed value of \( a > 0 \), define for \( \text{Re} \ s > 0 \) the function

\[
F(s) = \int_{0}^{1} x^s (1-x)^a \frac{dx}{x(1-x)}.
\]

Prove \( F(s) = \Gamma(a)\Gamma(s)/\Gamma(a + s) \). (Hint: show \( G(s) = F(s)\Gamma(a + s) \) satisfies the functional equation \( G(s + 1) = sG(s) \).)

6. Using the previous result, evaluate \( \int_{0}^{1} (1-x^a)^b \) for \( a, b > 0 \).

7. State and prove a necessary and sufficient condition for a meromorphic 1-form \( \omega = \omega(z) \, dz \) on \( C \) to be the logarithmic derivative, \( \omega = d\log f = f'(z)/f(z) \, dz \), of a meromorphic function \( f(z) \).

8. Prove:

\[
\frac{\coth(\pi)}{1^7} + \frac{\coth(2\pi)}{2^7} + \frac{\coth(3\pi)}{3^7} + \ldots = \frac{19\pi^7}{56700}.
\]

(Hint: you may use the fact that the residue of \( \cot(\pi z)\coth(\pi z)/z^7 \) at \( z = 0 \) is \( -19\pi^6/14175 \).)