DIFFERENTIAL GEOMETRY SEMINAR
Regularisation and meromorphic continuation of residue currents
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ABSTRACT

Let $X$ be a complex manifold of some dimension $n \geq 2$. Let $(f_1, \ldots, f_k)$ be a $k$-tuple in $\mathcal{O}(X)$ in complete intersection, i.e. $\{f_1 = \ldots = f_k = 0\}$ has codimension $k$. There exists the Coleff-Herrera current

$$\bar{\partial} \left( \frac{1}{f_1} \right) = \bar{\partial} \left( \frac{1}{f_1} \right) \wedge \left( \bar{\partial} \left( \frac{1}{f_k} \right) \right)$$

A natural procedure to obtain (i) as a limit of smooth $(0, k)$-forms is to define

$$\gamma_{\epsilon} = \bar{\partial} \left( \frac{1}{f_1^2 + \epsilon_1} \right) \wedge \ldots \wedge \bar{\partial} \left( \frac{1}{f_k^2 + \epsilon_k} \right)$$

for each $k$-tuple $\epsilon > 0$. It turns out that $\gamma_{\epsilon}$ has an unrestricted limit as the $p$-tuple $\epsilon \to 0$ which equals the Coleff-Herrera current (i). More precisely, to each $X_0 \subset X$ there exists an integer $M$, a constant $C$ and some $\delta > 0$ such that for every test-form $\Psi$ of bi-degree $(n, n-k)$ with support in $X_0$ one has

$$|\bar{\partial} \left( \frac{1}{f} \right)(\Psi) - \gamma_{\epsilon}(\Psi)| \leq C_M \cdot ||\Psi||_{M} \cdot (\epsilon_1 + \ldots + \epsilon_k)^{\delta}$$

Here $||\Psi||_{M}$ denotes the $C^M$-norm of the test-form.

Meromorphic extensions With $\lambda_1, \ldots, \lambda_k$ in $\mathbb{C}^k$ we consider the current-valued function

$$F(\lambda_1, \ldots, \lambda_k) = \bar{\partial} \left( \frac{1}{f_1^{\lambda_1}} \right) \wedge \ldots \wedge \bar{\partial} \left( \frac{1}{f_k^{\lambda_k}} \right)$$

Theorem: $F$ has a holomorphic extension to a domain in the $k$-dimensional complex $\lambda$-space which contains $\cap \{ \lambda_{\nu} \} > -\delta$ for some $\delta > 0$.

Remark: The case $k = 3$ was recently been established by Hkan Samuelson at Chalmers University in Sweden. Adapting Samuelson’s methods we prove the result for every $k \geq 3$. Let us remark that this existence of an unrestricted limit is remarkable in view of the counter-example due to A. Tsikh and M. Passare which shows that one cannot perform an unrestricted limit during the construction of the Coleff-Herrera current.