1. For each of the following functions, graph the function and its derivative on a graphing calculator. Then determine the number of critical points of the function.

   (a) \( f(x) = \frac{1}{3}x^3 + x + 1 \)
   (b) \( g(x) = \frac{1}{3}x^3 + 1 \)
   (c) \( h(x) = \frac{1}{4}x^3 - \frac{3}{2}x^2 + 1 \)

2. How many critical points can a generic cubic function \( f(x) = ax^3 + bx^2 + cx + d \) have? Justify your answer.

3. How many local extrema can a cubic function have? Justify your answer.

4. How many inflection points can a cubic function have? Justify your answer.

5. If \( f(x) = ax^3 + bx^2 + cx + d \), what are \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \)?

6. How many \( x \)-intercepts can a cubic function have? Justify your answer.

7. Sketch the graph of a cubic function \( f \) that ...

   (a) .. has roots at \( x = -5 \), \( x = 2 \), and \( x = 6 \).
   (b) .. has roots at \( x = -1 \) and \( x = 1 \) only and satisfies \( f(0) = 3 \).
   (c) .. has a roots at \( x = 3 \) only and satisfies \( \lim_{x \to \infty} f(x) = -\infty \).
   (d) .. has a local minimum at \( x = -1 \) and a local maximum at \( x = 3 \).
   (e) .. has an inflection point at \( x = 3 \).

8. Find an algebraic formula for each of the cubic functions described in Question 7.