Math 55a: Honors Advanced Calculus and Linear Algebra

Homework Assignment #8 (8 November 2002):
Linear Algebra IV — “Eigenstuff”

HINT, n.: The hardest of several possible ways to do a proof.¹

1.–10. Solve Exercises 4, 7–12, 15, 16, 21 from Chapter 5 of the textbook (pages 94,95). As usual, \( F \) can be any field, and \( C \) can be any algebraically closed field. Do not assume that vector spaces are finite dimensional unless you must. For #4, remember that Axler’s “null (\( T \))” is our “ker (\( T \))”. For #16, how much of #15 remains true over an arbitrary field?

[#15 has the following important consequence: if \( P \in F[z] \) and \( P(T) = 0 \) for some linear operator \( T \in L(V) \), then every eigenvalue of \( T \) is a root of \( P \). For instance, the only possible eigenvalues of a linear involution are \( \pm 1 \), the roots of \( z^2 - 1 \).]

For the next computational problem, make sure to check your answer against the actual entries of \( A^t \) for the first few \( t \).

11. Let \( A \) be the \( 2 \times 2 \) matrix \( \begin{bmatrix} 1 & 7 \\ 6 & 3 \end{bmatrix} \). Find a closed form for (the entries of) \( A^t \) as functions of \( t = 0,1,2,\ldots \). [Hint: Begin by finding the eigenvalues and eigenvectors of the linear transformation corresponding to \( A \).] What happens to \( A^t \) asymptotically as \( t \to \infty \)? What happens if \( A \) is replaced by the matrix \( \begin{bmatrix} 0.2 & 1.2 \\ -0.6 & 1.4 \end{bmatrix} \)?

This problem set is due Monday [sic], 18 November, at the beginning of class.

¹Definitions of Terms Commonly Used in Higher Math, R. Glover et al.; cf. also Prob. 11.