VECTORS. Two points \( P_1 = (x_1, y_1, z_1) \), \( Q = P_2 = (x_2, y_2, z_2) \) determine a vector \( \vec{v} = (x_2 - x_1, y_2 - y_1, z_2 - z_1) \). It points from \( P_1 \) to \( P_2 \) and we can write \( P_1 + \vec{v} = P_2 \).

COORDINATES. Points \( P \) in space are in one to one correspondence to vectors pointing from 0 to \( P \). The numbers \( \vec{v}_i \) in a vector \( \vec{v} = (v_1, v_2, v_3) \) are also called components or of the vector.

REMARKS: vectors can be drawn everywhere in space. If a vector starts at 0, then the vector \( \vec{v} = (v_1, v_2, v_3) \) points to the point \( (v_1, v_2, v_3) \). That’s is why one can identify points \( P = (a, b, c) \) in space with a vector \( \vec{v} = (a, b, c) \). Two vectors which are translates of each other are considered equal.

**ADDITION SUBTRACTION, SCALAR MULTIPLICATION.**

\[
\vec{u} + \vec{v} = (u_1, u_2, u_3) + (v_1, v_2, v_3) = (u_1 + v_1, u_2 + v_2, u_3 + v_3) \\
\vec{u} - \vec{v} = (u_1, u_2, u_3) - (v_1, v_2, v_3) = (u_1 - v_1, u_2 - v_2, u_3 - v_3) \\
\lambda \vec{u} = \lambda (u_1, u_2, u_3) = (\lambda u_1, \lambda u_2, \lambda u_3)
\]

**BASIS VECTORS.** The vectors \( \vec{i} = (1, 0, 0) \), \( \vec{j} = (0, 1, 0) \) and \( \vec{k} = (0, 0, 1) \) are called standard basis vectors.

Every vector \( \vec{v} = (v_1, v_2, v_3) \) can be written as a sum of standard basis vectors: \( \vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k} \).

WHERE DO VECTORS OCCUR? Here are some examples:

**Velocity** (see later): if \( (f(t), g(t)) \) is a point in the plane which depends on time \( t \), then \( \vec{v} = (f'(t), g'(t)) \) is the velocity vector at the point \( (f(t), g(t)) \).

**Forces:** Some problems in statics involve the determination of forces acting on objects. Forces are represented as vectors.

**Fields:** fields like electromagnetic or gravitational fields or velocity fields in fluids are described with vectors.

**Qbits:** in quantum computation, one does not work with bits, but with qbits, which are vectors.

**Color** can be written as a vector \( \vec{v} = (r, g, b) \), where \( r \) is red, \( g \) is green and \( b \) is blue. An other coordinate system is \( \vec{v} = (c, m, y) = (1 - r, 1 - g, 1 - b) \), where \( c \) is cyan, \( m \) is magenta and \( y \) is yellow.

**SVG.** Scalable Vector Graphics is an emerging standard for the web for describing two-dimensional graphics in XML.
VECTOR OPERATIONS. The addition and scalar multiplication of vectors satisfy "obvious" properties.

There is no need to memorize them.

We write * here for multiplication with a scalar but usually, the multiplication sign is left out.

TRIANGLE INEQUALITY: \( |v| + |w| \geq |v + w| \)

ORTHOGONAL VECTORS. Two vectors are called orthogonal if \( v \cdot w = 0 \). The zero vector \( \vec{0} \) is orthogonal to any vector. EXAMPLE: \( \vec{v} = (2, 3) \) is orthogonal to \( \vec{w} = (-3, 2) \).

PROJECTION. The vector \( \vec{a} = \text{proj}_{\vec{w}}(\vec{v}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|^2} \) is called the projection of \( \vec{v} \) onto \( \vec{w} \).

The scalar projection is defined as \( \text{comp}_{\vec{w}}(\vec{v}) = \frac{(\vec{v} \cdot \vec{w})}{|\vec{w}|} \). (Its absolute value is the length of the projection of \( \vec{v} \) onto \( \vec{w} \).) The vector \( \vec{b} = \vec{v} - \vec{a} \) is called the component of \( \vec{v} \) orthogonal to the \( \vec{w} \)-direction.

EXAMPLE. \( \vec{v} = (0, -1, 1), \vec{w} = (1, -1, 0) \), proj\(_{\vec{w}}(\vec{v}) = (1/2, -1/2, 0) \), comp\(_{\vec{w}}(\vec{v}) = 1/\sqrt{2} \).