Homework 11: One More Problem – Solutions

For each of the following surfaces:

(i) Decide whether the surface is the graph of a function \( f(x, y) \). If so, give a possible \( f(x, y) \). If not, explain why not.

(ii) Decide whether the surface is a level set of a function \( F(x, y, z) \). If so, give a possible \( F(x, y, z) \). If not, explain why not.

(iii) Decide whether the surface has a parametric representation. If so, give a possible \( r(u, v) \). If not, explain why not.

Here are the surfaces:

(a) \( z = x^2 + y^2 \)

Solution:

(i) This is a graph of \( z = f(x, y) = x^2 + y^2 \).

(ii) This is a level set: \( F(x, y, z) = z - x^2 - y^2 = 0 \).

(iii) This is a surface with a simple parametric representation: \( r(x, y) = (x, y, x^2 + y^2) \).

(b) \( x = y^2 + z^2 \)

Solution:

(i) This is not the graph of a function.

(ii) This is a level set: \( F(x, y, z) = x - y^2 - z^2 = 0 \).

(iii) This is a surface with a simple parametric representation: \( r(y, z) = (y^2 + z^2, y, z) \).

(c) \( 2x + 3y + 4z = 12 \)

Solution:

(i) This is a graph of \( z = f(x, y) = \frac{1}{4}(12 - 2x - 3y) \).

(ii) This is a level set: \( F(x, y, z) = 2x + 3y + 4z - 12 = 0 \).

(iii) This is a surface with a simple parametric representation: \( r(x, y) = (x, y, \frac{1}{4}(12 - 2x - 3y)) \).

There are, of course, many others. This one can be re-written as

\[
  r(x, y) = \langle 3, 0, 0 \rangle + x\langle 1, 0, -\frac{1}{2} \rangle + y\langle 0, 1, -\frac{3}{4} \rangle
\]

which in turn can be written more simply as

\[
  r(u, v) = \langle 3, 0, 0 \rangle + u\langle 2, 0, -1 \rangle + v\langle 0, 4, -3 \rangle.
\]