r(u, v) = \cos v i + \sin v j + u k. The parametric equations for the surface are x = \cos v, y = \sin v, z = u. Then x^2 + y^2 = \cos^2 v + \sin^2 v = 1 and z = u with no restriction on u, so we have a circular cylinder, graph IV. The grid curves with u constant are the horizontal circles we see in the plane z = u. If v is constant, both x and y are constant with v free to vary, so the corresponding grid curves are the lines on the cylinder parallel to the z-axis.

10. r(u, v) = u \cos v i + u \sin v j + v k. The parametric equations for the surface are x = u \cos v, y = u \sin v, z = v. Then x^2 + y^2 = u^2 \cos^2 v + u^2 \sin^2 v = u^2, which represents the equation of a cone with axis the z-axis, graph V. The grid curves with u constant are the horizontal circles we see, corresponding to the equations x^2 + y^2 = u^2 in the plane z = u. If v is constant, x, y, z are each scalar multiples of u, corresponding to the straight line grid curves through the origin.

11. r(u, v) = u \cos v i + v j + u k. The parametric equations for the surface are x = u \cos v, y = v, z = u. We look at the grid curves first; if we fix v, then x and y parametrize a straight line in the plane z = v which intersects the z-axis. If x is held constant, the projection onto the xy-plane is circular; with x = v, each grid curve is a helix. The surface is a spiraling ramp, graph I.

12. r(u, v) = u \cos v i + u \sin v j + v k. The parametric equations for the surface are x = u \cos v, y = u \sin v, z = v. Then x^2 + y^2 = u^2 \cos^2 v + u^2 \sin^2 v = u^2, so if u is held constant, each grid curve is a circle of radius u in the plane x = u^2. The graph then must be graph III. If v is held constant, so v = 0, we have y = u \sin 0 and x = u \cos 0. Then y = (\tan 0) x, so the grid curves we see running lengthwise along the surface in the planes y = kx correspond to keeping v constant.

13. x = (1 - \sin u) \cos v, y = (1 - \cos u) \sin v, z = u. If u is held constant, x and y give an equation of an ellipse in the plane z = u, thus the grid curves are horizontally oriented ellipses. Note that when u = 0, the “ellipse” is the single point (0, 0, 0), and when u = \pi, we have y = 0 while x ranges from -\pi to \pi, a line segment parallel to the x-axis in the plane z = \pi. This is the upper “seam” we see in graph II. When v is held constant, z = u is free to vary, so the corresponding grid curves are the curves we see running up and down along the surface.

14. x = (1 - u)(3 + \cos v) \cos 4u, y = (1 - u)(3 + \cos v) \sin 4u, z = 3u + (1 - u) \sin v. These equations correspond to graph VI: when u = 0, then x = 3 + \cos v, y = 0, and z = \sin v, which are equations of a circle with radius 1 in the xz-plane centered at (3, 0, 0). When u = \frac{1}{2}, then x = \frac{3}{2} + \frac{1}{2} \cos v, y = 0, and z = \frac{3}{2} + \frac{1}{2} \sin v, which are equations of a circle with radius \frac{1}{2} in the xy-plane centered at (\frac{3}{2}, 0, \frac{3}{2}). When u = 1, then x = y = 0 and z = 3, giving the topmost point shown in the graph. This suggests that the grid curves with u constant are the vertically oriented circles visible on the surface. The spiralling grid curves correspond to keeping v constant.

From Example 3, parametric equations for the plane through the points (1, 2, -3) that contains the vectors a = (1, 1, -1) and b = (1, -1, 1) are x = 1 + u(1) + v(1) = 1 + u + v, y = 2 + u(1) + v(-1) = 2 + u - v, z = -3 + u(-1) + v(1) = -3 - u + v.

(a) Here z = a \sin \alpha, y = |AB|, and x = |OA|. But |OB| = |OC| + |CB| = b + a \cos \alpha and \sin \theta = |\overrightarrow{AB}| |\overrightarrow{OB}| so that y = |\overrightarrow{OB}| \sin \theta = (b + a \cos \alpha) \sin \theta. Similarly \cos \theta = |\overrightarrow{OA}| |\overrightarrow{OB}| so z = (b + a \cos \alpha) \cos \theta. Hence a parametric representation for the torus is x = b \cos \theta + a \cos \alpha \cos \theta, y = b \sin \theta + a \cos \alpha \sin \theta, z = a \sin \alpha, where 0 \leq \alpha \leq 2\pi, 0 \leq \theta \leq 2\pi.

cylindrical coordinates, parametric equations are x = \sin \theta, y = y, z = \cos \theta, 0 \leq \theta \leq 2\pi, -1 \leq y \leq 3.