18. (a) \( f_x(x, y, z) = 2xz + y^2 \) implies \( f(x, y, z) = x^2z + xy^2 + g(y, z) \) and so \( f_y(x, y, z) = 2xy + g_y(y, z) \). But \( f_y(x, y, z) = 2xy \) so \( g_y(y, z) = 0 \) \( \Rightarrow \) \( g(y, z) = h(z) \). Thus \( f(x, y, z) = x^2z + xy^2 + h(z) \) and \( f_x(x, y, z) = x^2 + h'(z) \). But \( f_x(x, y, z) = x^2 + 3z^2 \), so \( h'(z) = 3z^2 \) \( \Rightarrow \) \( h(z) = x^3 + K \). Hence \( f(x, y, z) = x^3z + xy^2 + z^3 \) (taking \( K = 0) \).

(b) \( t = 0 \) corresponds to the point \((0, 1, -1)\) and \( t = 1 \) corresponds to \((1, 2, 1)\), so
\[
\int_C \mathbf{F} \cdot d\mathbf{r} = f(1, 2, 1) - f(0, 1, -1) = 6 - (-1) = 7.
\]

26. \( \nabla f(x, y) = \cos(x - 2y)\mathbf{i} - 2\cos(x - 2y)\mathbf{j} \)

(a) We use Theorem 2: \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_1} \nabla f \cdot d\mathbf{r} = f(r(b)) - f(r(a)) \) where \( C_1 \) starts at \( a \) and ends at \( b \).

So because \( f(0, 0) = \sin 0 = 0 \) and \( f(\pi, \pi) = \sin(\pi - 2\pi) = 0 \), one possible curve \( C_1 \) is the straight line from \((0, 0)\) to \((\pi, \pi)\); that is, \( r(t) = \pi t \mathbf{i} + \pi t \mathbf{j}, 0 \leq t \leq 1 \).

(b) From (a), \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = f(r(b)) - f(r(a)) \). So because \( f(0, 0) = \sin 0 = 0 \) and \( f(\frac{\pi}{2}, 0) = 1 \), one possible curve \( C_2 \) is \( r(t) = \frac{\pi}{2} t \mathbf{i}, 0 \leq t \leq 1 \), the straight line from \((0, 0)\) to \((\frac{\pi}{2}, 0)\).

8. The region \( D \) enclosed by \( C \) is given by \( \{(x, y) \mid 0 \leq x \leq 1, 3x \leq y \leq 3\} \), so
\[
\int_C x^2y^2 \, dx + 4xy^3 \, dy = \iint_D \left( \frac{\partial}{\partial x} (4xy^3) - \frac{\partial}{\partial y} (x^2y^2) \right) \, dA
= \int_0^1 \int_{3x}^3 (4y^3 - 2x^2y) \, dy \, dx
= \int_0^1 \left[ y^4 - x^2y^2 \right]_{3x}^3 \, dx
= \int_0^1 (81 - 9x^2 - 72x^3) \, dx
= 81 - 3 - \frac{72}{4} = \frac{39}{4}.
\]

19. By Green's Theorem, \( W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_D (x^2 + 3xy^2) \, dy = \iint_D (3x^2 + 3y^2 - 0) \, dA \), where \( Q \) is the semicircular region bounded by \( C \). Converting to polar coordinates, we have
\[
W = 3 \int_0^{\pi/2} \int_0^1 r^3 \cdot r \, dr \, d\theta = 3\pi \left[ \frac{1}{4} r^4 \right]_0^1 = 12\pi.
\]

20. \( A = \int_C x \, dy = \int_0^{2\pi} (\cos t) (3 \sin^3 t \cos t) \, dt = 3 \int_0^{2\pi} \frac{1}{4} (1 - \cos 4t) \, dt = \frac{3}{2} \pi \).