Introduction

Dfield & pplane are programs designed for phase plane analysis of differential equations. Dfield is used on first order differential equations of the form \( x' = f(t,x) \), while pplane is used for a system of differential equations of the form \( x' = f(x,y) \), \( y' = g(x,y) \). As pplane includes all functions available in dfield, a discussion of pplane will be sufficient.

Getting Started

Free Java version is available at http://math.rice.edu/~dfield/dfpp.html

Browser Specifications
Java 1.1.6 enabled
• PCs: Netscape 6 works best, but IE works too (you do need to configure for printing)
• Macs: only works on Mozilla

When the program first starts, it opens 4 windows: PPLANE Equation Window, PPLANE Messages, PPLANE Phase Plane, and a PPLANE copyright window. Click OK to continue.

• PPLANE Equation Window: here you enter in your equations, specify parameters and set window size for your graph
• PPLANE Messages: displays coordinates, possible solutions, eigenvalues, etc. when you calculate orbits or equilibrium points
• PPLANE Phase Plane: the actual graph. You can graph nullclines, trajectories (aka orbits), \( x(t) \) vs \( t \), find equilibrium points, etc.

Fig. 1: windows at the start of pplane
Graphing Systems of Differential Equations

Fig. 2: the PPLANE Equation Window showing how to graph a system of differential equations.

Entering in the Equations
Type in your equations at the top part of the PPLANE Equation Window. You can use whatever variables you want; there’s no need to stick to x & y. You’ll get red error messages as you’re typing in the equation—ignore them.

Defining Parameters
You may have constants in your equations (e.g. a for the system of equations describing populations of right-handed & left-handed snails). Be sure to define them in the “Parameter Expressions” section of the PPLANE Equation Window. This is useful because you can easily study several different systems (e.g. snail populations for different a values).

Changing the Display Window
In the “Display Window” section of the PPLANE Equation Window, you can define the range of x & y values you want to graph. It’s pretty self-explanatory.

Graphing
Finally, click **Graph Phase Plane** to graph your system.
Graphing Trajectories/Orbits

Note: Everything from now on refers to the PPLANE Phase Plane window unless specified otherwise.

Changing the Slope Field
From the Options menu, select Direction Field Settings to change the number of rows and columns, the way the field is displayed and computation settings.

Graphing solution curves (trajectories/orbits)
Click on the point from which you want a trajectory. The PPLANE Messages window will display the point you clicked on and any possible equilibrium points or solutions nearby.

Erasing Orbits
Go to Edit and select Delete Orbit or Delete All Orbits

Changing directions
To specify whether you want to graph the orbit in the forward (t>0), backward (t<0), or both directions, go to Options then Solution Direction to choose.

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Fig. 3: An orbit going in both directions; the message associated with that orbit, and the direction field setting box.
Changing time delay

You can make the orbit move slower so you can watch its motion. Go to Options and click on the desired speed at Delay Time per Point.

Inputting the initial value

To manually choose the starting point for a solution curve, go to the Solution menu and click on Keyboard Input of Initial Value. Enter in the initial value and then click Solve.

Other options

• Points
  • From the Options menu, click on Show Points to show each point on an orbit.
• Cross hairs
  • To make it easier to see the coordinates of your mouse (as you move it around the graph), click on Show cross-hairs in the Options menu.
• Zoom
  • To zoom in, go to Edit and pick the zoom you want.

Fig. 4: Initial value input box.

Fig. 5: (left) Options menu & orbit with points shown. (right): Crosshairs give the location of the mouse.
Analyzing Stability

Graphing nullclines
From the Solutions menu, click Show Nullclines for just the nullclines or Show Nullclines + Arrows for the arrows as well.

Finding equilibrium points
You can find equilibrium points by selecting Find an Equilibrium Point from the Solutions menu and clicking at a point on the graph. The coordinates of the point, the Jacobian, eigenvalues and eigenvectors for the point are displayed in the PPLANE Messages window. An easy way to view the equilibrium points is to click on Equilibrium Point Linearization in the Solutions menu.

Fig. 6: (right) a graph of the nullclines with arrows. Equilibrium points are displayed. (bottom) the PPLANE Messages window displaying information about the equilibrium point.

Graphing x vs t
From the Graph menu, select x vs. t or whatever option you want. Then click on the desired solution curve, and a window with the graph should pop up. The vertical lines made by clicking on the graph can be erased by clicking Restore.

Fig. 7: a graph of x vs. t and y vs. t
Other Stuff

Printing
Click **File** and then **Print**. Note that IE users may need configuration before printing:

• From the Tools menu choose "Internet Options".
• Click the button labeled "Security".
• Near the bottom, click the button labeled "Custom Level ...".
• Click the button labeled "Java Custom Settings" on the bottom left of the screen.
• Click the tab labeled "Edit Permissions".
• In the menu look for "Unsigned Content --> Run Unsigned Content --> Additional Unsigned Permissions --> Printing". Under this heading, click "Enable.".
• Exit out of all of the windows, making sure that your new option is activated.
• Restart your computer.

Templates of Common Differential Equation Systems
In the PPLANE Equations Window, the **Gallery** menu contains many common systems of differential equations such as the predator-prey system. You can even save other systems (e.g. the right-handed & left-handed snails model) in the gallery.

That’s all Folks
That pretty much covers the basics of pplane that you’ll need. The best way to learn pplane is just to play around with it. You’ll find that it’s pretty simple.

If you have any other questions, don’t hesitate to ask me, Ana or Prof. Judson for help.

![Fig. 8: the gallery menu](image)