Homework 5
Math 124, Fall 2005
Due Wednesday, October 26

No late assignments will be accepted as solutions will be posted on Thursday morning.

1. Express the following numbers as sums of two squares or prove that it is not possible. Show all your work.
   \[97 \ 221 \ 300 \ 490 \ 729 \ 1001 \ 490490\]

2. Use Serret’s construction to write 449 as a sum of two squares. [Hint: \(67^2 \equiv -1 \mod 449\).]

3. (a) Prove that the following equation has a solution.
   \[h^2 \equiv -5 \mod 23\]
   (b) Prove that the following equation has no solutions.
   \[23 = x^2 + 5y^2\]
   (c) Conclude that they are more than one equivalence classes of quadratic forms of discriminant -5.

4. (a) Prove that there are two equivalence classes of quadratic forms of discriminant -12, corresponding to the following two reduced forms.
   \[x^2 + 3y^2 \quad 2x^2 + 2xy + 2y^2\]
   (b) Show that a prime \(p\) is representable by \(x^2 + 3y^2\) if and only if \(p = 2\) or \(p \equiv 2 \mod 6\).