Homework 4
Math 124, Fall 2004
Due Wednesday, October 19

No late assignments will be accepted as solutions will be posted on Thursday morning. Good luck!

Throughout this assignment we will denote by

\[ < q_0, q_1, \ldots, q_n > = q_0 + \frac{1}{q_1 + \frac{1}{q_2 + \frac{1}{q_3 + \ldots}}}. \]

1. Find all primes \( p \) such that the equation

\[ x^2 \equiv 13 \mod p \]

has a solution.

2. (a) Evaluate the following infinite continued fractions.
   i. \( < 2, 3, 1, 1, 1, 1, 1, \ldots > \)
   ii. \( < 1, 3, 1, 2, 1, 2, 1, 2, 1, \ldots > \)

(b) Expand \( \sqrt{5} \) and \( \sqrt{20} \) as continued fractions.

(c) Find the continued fraction for the number \( \sqrt{n(n + 1)} \).

3. (a) Given positive integers \( d < c \) show that

\[ < a, c > < < a, d > \]
\[ < a, b, c > > < a, b, d > \]

(b) Let \( a_1, \ldots, a_n \) and \( c \) be positive real numbers. Prove that

\[ < a_0, a_1, \ldots, a_n + c > < < a_0, \ldots, a_n > \quad n = 2k + 1 \]
\[ < a_0, a_1, \ldots, a_n + c > > < a_0, \ldots, a_n > \quad n = 2k \]

(c) Let \( a_0, \ldots, a_n \) and \( b_0, \ldots, b_{n+1} \) be positive integers. State the conditions for

\[ < a_0, \ldots, a_n > < < b_0, \ldots, b_n, b_{n+1} > \]. \]