Name: ____________________________

In Class MidTerm Exam for Math 113
10 - 11:30 am, March 20, 2008

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• Please show ALL your work on this exam paper. Partial credit will be awarded where appropriate.

• Some problems will be asking you to prove results covered in class. For those that do not, you may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.

• NO calculators are permitted.
1. (15 points) Simplify the following expressions:
   
   (a) $e^{\log i}$
   (b) $\log i$
   (c) $i^{\log(-1)}$
2. (15 points)
   (a) Prove that the $n$th roots of unity can be expressed as $1, w, w^2, w^3, \ldots, w^{n-1}$.
   (b) What is $w$?
   (c) Show that the sum of the $n$th roots of unity is zero.
3. (a) (7 points) Is $f(z) = \overline{z}$ analytic? Prove or disprove.

(b) (7 points) Let $A$ be an open subset of $\mathbb{C}$ and $A^* = \{ z : \overline{z} \in A \}$. Suppose $f$ is analytic on $A$ and define a function $g$ on $A^*$ by

$$g(z) = f(\overline{z}).$$

Show that $g$ is analytic on $A^*$. 
4. (15 points)

(a) Define what it means for a function to be analytic (holomorphic) at a point $z \in \mathbb{C}$.
(b) State the Cauchy-Riemann equations.
(c) Prove that a function analytic at a point $z \in \mathbb{C}$ satisfies the Cauchy-Riemann equations at that point.
5. Evaluate the following integrals with \( \gamma \) being the unit circle centered at the origin:

(a) (3 points) \( \int_{\gamma} \sin z \, dz \).

(b) (3 points) \( \int_{\gamma} \frac{\sin z}{z} \, dz \).

(c) (3 points) \( \int_{\gamma} \frac{\sin z}{z^2} \, dz \).

(d) (3 points) \( \int_{\gamma} \frac{\sin(e^z)}{z^2} \, dz \).
6. (14 points) Evaluate \( \int_0^{2\pi} e^{-i\theta} e^{i\theta} d\theta \).
7. (15 points) Let \( f \) be entire and let \( |f(z)| \leq M \) for \( z \) on the circle \( |z| = R \) with \( R \) fixed. Prove that

\[
|f^{(k)}(z)| \leq \frac{k!M}{(R - r)^k}, \quad r = |z|,
\]

for all \( 0 \leq r < R \) and for all \( k = 0, 1, 2, 3, \ldots \)
Extra space for work: