1) A Dirichlet series is a series \( D(s) = \sum_{1 \leq n < \infty} a_n n^{-s} \), with complex coefficients \( a_n \) such that \( |a_n| = O(n^N) \) for some \( N \in \mathbb{Z} \). Note that the series converges absolutely and locally uniformly for \( s \in \mathbb{C}, \text{Re}\ s > N + 1 \). The least real number \( \sigma \) such that the series converges absolutely for \( \text{Re}\ s > \sigma \) (so \( \sigma \leq N + 1 \)) is called the abscissa of absolute convergence. Show: for \( \sigma_1 > \sigma \) and all \( n \geq 1 \),

\[
    a_n = \lim_{T \to \infty} \frac{n^{\sigma_1}}{T} \int_0^T D(\sigma_1 + it) n^i dt.
\]

Deduce that \( D(s) \) cannot vanish identically unless all the coefficients \( a_n \) vanish. Can this last conclusion be seen more directly?

2) The point of this problem is to identify the genus of the compactification of \( G_\mathbb{Z} \backslash H \) without using topology or differential geometry. It is a little easier to do so for \( \Gamma(2) \backslash H \), since \( \Gamma(2) \) is the principal congruence subgroup of level 2 acts on \( H \) without fixed points, and then to relate the genus of the compactification of \( \Gamma(2) \backslash H \) to that of the compactification of \( G_\mathbb{Z} \backslash H \).

a) In class, we proved that for any subgroup of finite index \( \Gamma \subset G_\mathbb{Z} \) of finite index, the compactification of \( \Gamma \backslash H \) has genus

\[
    g = 1 + \frac{n}{12} - \frac{n_2}{4} - \frac{n_3}{3} - \frac{n_\infty}{2},
\]

with \( n = \) number of sheets of the ramified covering \( \Gamma \backslash H \to G_\mathbb{Z} \backslash H \), \( n_j = \) number of fixed points of order \( j \), \( j = 2 \) or \( j = 3 \), and \( n_\infty = \) number of cusps. This formula was derived using the fact that the compactification of \( G_\mathbb{Z} \backslash H \) has genus 0; if, for the purposes of this problem, the genus is some unknown integer \( g_0 \), how does the above formula need to be modified?

b) Determine the integers \( n, n_2, n_3 \) and \( n_\infty \) for \( \Gamma = \Gamma(2) \).

c) In Math 213a, we used purely analytic methods to identify the universal covering of \( \mathbb{C} - \{0,1\} \) with \( H \) and the universal covering group with \( \Gamma(2)/\{\pm 1\} \). Using this result and parts a), b) to determine the genus of the compactification of \( G_\mathbb{Z} \backslash H \).