1) Show that the representation \((\ell, V_k)\) defined in class is strongly continuous – i.e., for every \(v \in V_k\), the function \(g \mapsto \ell(g)v\) is continuous, as map from \(G\) to \(V_k\).

2a) Show that the hyperbolic Laplacian \(\Delta = y^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)\) on \(H\) is invariant under the action of \(G = SL(2, \mathbb{R})\).

b) Prove that the so-called non-holomorphic Eisenstein series
\[
E_s(z) = \sum_{\gamma \in \Gamma \setminus \Gamma} (\operatorname{Im} \gamma z)^s, \quad \text{with} \quad \operatorname{Re} s > 1,
\]
is well defined and converges locally uniformly (in both \(z\) and \(s\)) to a \(\Gamma\)-invariant eigenfunction of \(\Delta\). Also show that \(E_s(z)\) has moderate growth in the variable \(z\) and depends holomorphically on the parameter \(s\). In doing this problem, you may use the following standard fact: on the kernel of an elliptic linear differential operator, such as \(\Delta - \lambda\), the topology of locally uniform convergence coincides with the \(C^\infty\) topology.

c) Expanding \(E_s(z)\) as a Fourier series in \(x\), show that
\[
E_s(z) = y^s + \sum_{n \in \mathbb{Z} - \{0\}} a_n(s) b_n(|y|) e(nx),
\]
where \(b_n \in C^\infty(\mathbb{R}_{>0})\) satisfies a certain second order differential equation and has moderate growth for both \(y \to \infty\) and \(y \to 0\).

d) Compute the coefficients \(a_n(s)\) and deduce that \(E_s(z)\) can be meromorphically continued to the entire \(s\)-plane.