

Mathematical Platonism and its Opposites

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We had the sky up there, all speckled with stars, and we used to lay on our backs and look up at them, and discuss about whether they was made or only just happened—Jim he allowed they was made, but I allowed they happened; I judged it would have took too long to make so many.

mused Huckleberry Finn. The analogous query that mathematicians continually find themselves confronted with when discussing their art with people who are not mathematicians is:

Is mathematics discovered or invented?

I will refer to this as *The Question*, acknowledging that this five-word sentence, ending in a question-mark—and phrased in far less contemplative language than that used by Huck and Jim—may open conversations, but is hardly more than a token, standing for puzzlement regarding the status of mathematics.

One thing is—I believe—incontestable: if you engage in mathematics long enough, you bump into The Question, and it won't just go away¹. If we wish to pay homage to the passionate felt experience that makes it so wonderful to think mathematics, we had better pay attention to it.

Some intellectual disciplines are marked, even scarred, by analogous concerns. Anthropology, for example has a vast, and dolefully introspective, literature dealing with the conundrum of whether we can ever avoid—wittingly or unwittingly—clamping the templates of our own culture onto whatever it is we think we are studying: how much are we discovering, how much inventing?

Such a *discovered/invented* perplexity may or may not be a burning issue for other intellectual pursuits, but it burns exceedingly bright for mathematics, and with a strangeness that isn't quite

¹Garrison Keillor, a wonderful radio raconteur has in his repertoire a fictional character, Guy Noir, who tangles indefatigably with “life's persistent questions.” This is all to the good. We should pay particular honor to the category of persistent questions even though—or, especially because—those are the chestnuts that we'll never crack.

matched when it pops up in other fields. For example, if you were to say—as Thomas Kuhn once did— “Priestley discovered oxygen but Lavoisier invented it” I think I know roughly what you mean by that utterance, without our having to synchronize our private vocabularies terribly much. But to intelligently comprehend each other’s possibly differing attitudes towards circles, triangles, and numbers, we would also have to come to some—albeit ever-so-sketchy—understanding of how we each view, and talk about, a lot more than mathematics².

For me, at least, the anchor of any conversation about these matters is the experience of doing mathematics, and of groping for mathematical ideas. When I read literature that is ostensibly about The Question, I ask myself whether or not it connects in any way with my felt experience, and even better, whether it reveals something about it. I’m often—perhaps always—disappointed. The bizarre aspect of the mathematical experience—and this is what gives such fierce energy to The Question—is that one feels (I feel) that mathematical ideas can be hunted down, and in a way that is essentially different from, say, the way I am currently hunting the next word to write to finish this sentence. One can be a hunter and gatherer of mathematical concepts, but one has no ready words for the location of the hunting grounds. Of course we humans are beset with illusions, and the feeling just described could be yet another. There may be no location.

There are at least two standard ways of—if not exactly answering, at least—fielding The Question by offering a vocabulary of *location*. The colloquial tags for these locations are *In Here* and *Out There* (which seems to me to cover the field).

The first of these standard attitudes, the one with the logo *In Here*—which is sometimes called the Kantian (poor Kant!)—would place the source of mathematics squarely within our faculties of understanding. Of course faculties (*Vermögen*) and understanding (*Verstand*) are loaded eighteenth century words and it would be good—in this discussion at least—to disburden ourselves of their baggage as much as possible. But if this camp had to choose between discovery and invention, those two too-brittle words, it would opt for *invention*.

The “Out There” stance regarding the discovery/invention question whose heraldic symbol is Plato (poor Plato!) is to make the claim, starkly, that mathematics is the account we give of the timeless architecture of the cosmos. The essential mission, then, of mathematics is the accurate description, and exfoliation, of this architecture. This approach to the question would surely pick *discovery* over invention.

Strange things tend to happen when you think hard about either of these preferences.

For example, if we adopt what I labeled the Kantian position we should keep an eye on the stealth word “our” in the description of it that I gave, hidden as it is among behemoths of vocabulary (*Vermögen*, *Verstand*). Exactly *whose* faculties are being described? Who is the *we*? Is the *we* meant to be each and every one of us, given our separate and perhaps differing and often faulty faculties? If you feel this to be the case, then you are committed to viewing the mathematical enterprise to be as variable as humankind. Or are you envisioning some sort of distillate of all actual faculties, a more transcendental faculty, possessed by a kind of universal or ideal *we*, in

²For a start: you and I turn adjectives into nouns (*red* cows \mapsto red; *five* cows \mapsto five) with only the barest flick of a thought. What *is* that flick? Understanding the differences in our sense of what is happening here may tell us lots about our differences regarding matters that can only be discussed with much more mathematical vocabulary.

which case the Kantian view would seem to merge with the Platonic³.

If we adopt the Platonic view that mathematics is discovered, we are suddenly in surprising territory, for this is a full-fledged theistic position. Not that it necessarily posits a god, but rather that its stance is such that the only way one can adequately express one's faith in it, the only way one can hope to persuade others of its truth, is by abandoning the arsenal of rationality, and relying on the resources of the prophets.

Of course, professional philosophers are in the business of formulating anti-metaphysical or meta-physical positions, decorticating them, defending them, and refuting them⁴. Mathematicians, though, may have another—or at least a prior—duty in dealing with The Question. That is, to be meticulous participant/observers, faithful to the one aspect of The Question to which they have sole proprietary rights: their own imaginative experience. What, precisely, describes our inner experience when we (and here the *we* is you and me) grope for mathematical ideas? We should ask this question open-eyed, allowing for the possibility that whatever it is we experience may delude us into fabricating ideas about some larger framework, ideas that have no basis⁵.

I suspect that many mathematicians are as unsatisfied by much of the existent literature about The Question as I am. To be helpful here, I've compiled a list of Do's and Don't's for future writers promoting the Platonic or the Anti-Platonic persuasions.

- **For the Platonists.** One crucial consequence of the Platonic position is that it views mathematics as a project akin to physics, Platonic mathematicians being—as physicists certainly are—*describers* or possibly *predictors*—not, of course, of the physical world, but of some other more noetic entity. Mathematics—from the Platonic perspective—aims, among other things, to come up with the most faithful description of that entity.

This attitude has the curious effect of reducing some of the urgency of that staple of mathematical life: *rigorous proof*. Some mathematicians think of mathematical proof as *the* certificate guaranteeing trustworthiness of, and formulating the nature of, the building-blocks of the edifices that comprise our constructions. Without proof: no building-blocks, no edifice. Our step-by-step articulated arguments are the devices that some mathematicians feel are responsible for bringing into being the theories we work in. This can't quite be so for the ardent Platonist, or at least it can't be so in the same way that it might be for the non-Platonist. Mathematicians often wonder about—sometimes lament—the laxity of proof in the physics literature. But I believe this kind of lamentation is based on a misconception, namely the misunderstanding of the fundamental function of *proof* in physics. Proof has principally (as

³A more general lurking question is exactly how we are to view the various ghosts in the machine of Kantian idealism—for example, who exactly *is* that little-described player haunting the elegant concept of *universally subjective judgments* and going under a variety of aliases: the *sensus communis* or the *allgemeine Stimme*?

⁴A very useful—and to my mind, fine—text that does exactly this type of lepidoptery is Mark Balaguer's *Platonism and Anti-Platonism in Mathematics*, Oxford Univ. Press (1998).

⁵When I'm working I sometimes have the sense—possibly the illusion—of gazing on the bare platonic beauty of structure or of mathematical objects, and at other times I'm a happy Kantian, marveling at the generative power of the intuitions for setting what an Aristotelian might call *the formal conditions of an object*. And sometimes I seem to straddle these camps (and this represents no contradiction to me). I feel that the intensity of this experience, the vertiginous imaginings, the leaps of intuition, the breathlessness that results from “seeing” but where the sights are of entities abiding in some realm of ideas, and the passion of it all, is what makes mathematics so supremely important for me. Of course, the realm might be illusion. But the experience?

it should have, in physics) a rhetorical role: to convince others that your description holds together, that your model is a faithful *re*-production, and possibly to persuade yourself of that as well. It seems to me that, in the hands of a mathematician who is a determined Platonist, proof could very well serve primarily this kind of rhetorical function—making sure that the description is on track—and not (or at least: not necessarily) have the rigorous theory-building function it is often conceived as fulfilling.

My feeling, when I read a Platonist’s account of his or her view of mathematics, is that unless such issues regarding the nature of proof are addressed and conscientiously examined, I am getting a superficial account of the philosophical position, and I lose interest in what I am reading.

But the main task of the Platonist who wishes to persuade non-believers is to learn the trade, from prophets and lyrical poets, of how to communicate an experience that transcends the language available to describe it. If all you are going to do is to chant credos synonymous with “the mathematical forms are out there,”—which some proud essays about mathematical Platonism content themselves to do—well, that will not persuade.

- **For the Anti-Platonists.** Here there are many pitfalls. A common claim, which is meant to undermine Platonic leanings, is to introduce into the discussion the theme of *mathematics as a human, and culturally dependent pursuit* and to think that one is actually conversing about the topic at hand. Consider this, though: If the pursuit were *writing a description of the Grand Canyon* and if a Navajo, an Irishman, and a Zoroastrian were each to set about writing their descriptions, you can bet that these descriptions will be culturally-dependent, and even dependent upon the moods and education and the language of the three describers. But my having just recited all this relativism regarding the three descriptions does not undermine our firm faith in the *existence* of the Grand Canyon, their common focus. Similarly, one can be the most ethno-mathematically conscious mathematician on the globe, claiming that all our mathematical scribing is as contingent on ephemeral circumstance as this morning’s rain, and *still* one can be the most devout of mathematical Platonists.

Now this pitfall that I have just described is harmless. If I ever encounter this type of *mathematics is a human activity* argument when I read an essay purporting to defuse, or dispirit, mathematical Platonism I think to myself: human activity! what else could it be? I take this part of the essay as being irrelevant to The Question.

A second theme that seems to have captured the imagination of some anti-Platonists is recent neurophysiological work— a study of blood flow into specific sections of the brain—as if this gives an *insider’s view* of things⁶. Well, who knows? Neuro-anatomy and chemistry have been helpful in some discussions, and useless in others. To show this theme to be relevant would require a precisely argued explanation of exactly how *blood flow patterns* can refute, or substantiate, a Platonist—or any—disposition. A satisfying argument of that sort would be quite a marvel! But just slapping the words *blood flow*—as if it were a poker-hand—onto a page doesn’t really work.

Sometimes the mathematical anti-Platonist believes that headway is made by showing Platonism to be unsupportable by rational means, and that it is an incoherent position to take when formulated in a propositional vocabulary.

It is easy enough to throw together propositional sentences. But it is a good deal more difficult to capture a Platonic disposition in a propositional formulation that is a full and

⁶like the old Woody Allen movie *Everything you wanted to know about sex but were afraid to ask*

honest expression of some flesh-and-blood mathematician's view of things. There is, of course, no harm in trying—and maybe its a good exercise. But even if we cleverly came up with a proposition that *is* up to the task of expressing Platonism formally, the mere fact that the proposition cannot be demonstrated to be true won't necessarily make it vanish. There are many things—some true, some false—unsupportable by rational means. For example, if you challenge me to support—by rational means—my claim that I dreamt of Waikiki last night, I couldn't.

So, when is there harm? It is when the essayist becomes a *leveller*. Often this happens when the author writes extremely well, super coherently, slowly withering away the Platonist position by—well—the brilliant subterfuge of making the whole discussion boring, until I, the reader, become convinced—albeit momentarily, within the framework of my reading the essay—that there is no “big deal” here: the mathematical enterprise is precisely like any other cultural construct, and there is a fallacy lurking in any claim that it is otherwise. The Question is a non-question.

But someone who is not in love won't manage to definitively convince someone in love of the nonexistence of eros; so this mood never overtakes me for long. Happily I soon snap out of it, and remember again the remarkable sense of independence—autonomy even—of mathematical concepts, and the transcendental quality, the uniqueness—and the passion—of doing mathematics. I resolve then that (Plato or Anti-Plato) whatever I come to believe about The Question, my belief *must* thoroughly respect and not ignore all this.