Whether or not you know any mathematics, reading Tobias Dantzig's ”Number: The Language of Science” is captivating. The book you hold in your hands is a many-stranded meditation on Number, and is an ode to the beauties of mathematics.

The first, and oldest section of this classic is entitled ”Evolution of the Number Concept.” Yes: Number has had, and will continue to have, an *evolution*. How did Number begin? We can only speculate.

Did Number make its initial entry into language as an adjective?

Three cows, three days, three miles. Imagine the exhilaration you would feel if you were the first human to be struck with the startling thought that a unifying thread binds “three cows” to ”three days,” and that it may be worthwhile to deal with their common three-ness. This, if it ever occurred to a single person at a single time, would have been a monumental leap forward, for the disembodied concept of three-ness, the noun *three*, embraces far more than cows, or days. It would also have set the stage for the comparison to be made between, say, one day and three days, thinking of the latter duration as triple the former, ushering in yet another view of *three*, in its role in the activity of tripling; *three* embodied, if you wish, in the verb: to triple.

Or perhaps Number emerged from some other route: an incantatory intonement, for example, as children do: ”one, two, buckle my shoe . . .. ”

However it began, this story is still going on, and Number, humble Number, is showing itself ever more central to our understanding of *what is*. The early Pythagoreans must be dancing in their caves.

If I were someone who had a yen to learn about math, but never had the time to do so, and if I found myself marooned on that proverbial ”desert island,” the one book I would hope to have along is, to be honest, a good swimming manual. But the second book might very well be this one. For Dantzig accomplishes these essential tasks of scientific exposition: to assume no more than a general educated background; to give a clear and vivid account of material most essential to the story being told; to tell an important story; and--the task most rarely achieved of all--to explain ideas and not merely allude to them.

One of the beautiful strands in the story of Number is the manner in which the concept changed as mathematicians expanded the republic of numbers: from the counting numbers

$$1,2,3,\ldots$$

to the realm that includes negative numbers, and zero;

…-3,-2,-1,0,+1,+2,+3,…

and then to fractions, real numbers, complex numbers; and, via a different mode of colonization, to infinity and the hierarchy of infinities. Dantzig brings out the motivation for each of these augmentations: there is indeed, a unity that ties these separate steps into a single narrative. In the midst of his discussion of the expansion of the number concept, Dantzig quotes Louis XIV, who when asked what the guiding principle was of his international policy answered “Annexation! One can always find a clever lawyer to vindicate the act.” But Dantzig himself doesn't relegate anything to legal counsel. He offers intimate glimpses of mathematical birth-pangs, while constantly focusing on the vital question that hovers over this story: ”What does it mean for a mathematical object to exist?” Dantzig, in his comment about the emergence of complex numbers muses that ”For centuries [the concept of complex numbers] figured as a sort of mystic bond between reason and imagination.” He quotes Leibniz to convey this turmoil of the intellect:

> [T]he Divine Spirit found a sublime outlet in that wonder of analysis, that portent of the ideal world, that amphibian between being and not-being, which we call the imaginary root of negative unity.

Dantzig also tells us of his own early moments of perplexity:

> I recall my own emotions: I had just been initiated into the mysteries of the complex number. I remember my bewilderment: here were magnitudes patently impossible and yet susceptible of manipulations which lead to concrete results. It was a feeling of dissatisfaction, of restlessness, a desire to fill these illusory creatures, these empty symbols, with substance. Then I was taught to interpret these beings in a concrete geometrical way. There came then an immediate feeling of relief, as though I had solved an enigma, as though a ghost which had been causing me apprehension turned out to be no ghost at all, but a familiar part of my environment.

The interplay between algebra and geometry is one of the grand themes of mathematics. The magic of high school analytic geometry that allows you to describe geometrically intriguing curves by simple algebraic formulas and tease out hidden properties of geometry by solving simple equations has flowered--in modern mathematics—into a powerful intermingling of algebraic and geometric intutions, each fortifying the other. René Descartes proclaimed: ”I would borrow the best of geometry and of algebra and correct all the faults of the one by the other.” The contemporary mathematician, Sir Michael Atiyah, in comparing the glories of geometric intuition with the extraordinary efficacy of algebraic methods, wrote recently:

> Algebra is the offer made by the devil to the mathematician. The devil says: I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvelous machine.

It takes the delicacy of Dantzig to tell of the millennia-long courtship between arithmetic and geometry without smoothing out the Faustian edges of this love story.

In Euclid's *Elements of Geometry* we encounter Euclid's "Definition 2. A line is breadthless length." Nowadays, we have other perspectives on that staple of plane geometry, the straight line. We have the number-line, represented as a horizontal straight line extended infinitely in both directions on which all numbers positive, negative, whole numbers, fractional, or irrational, have their position. Also, to picture time variation, we call upon that crude model, the time-line, again represented as a horizontal straight line extended infinitely in both directions, to stand for the profound, ever-baffling ever-moving frame of past/present/futures that we think we live in. The story of how these different conceptions of *straight line* negotiate with each other is yet another strand of Dantzig's tale.

Dantzig truly comes into his own in his discussion of the relationship between time and mathematics. He contrasts Cantor's theory, where infinite processes abound, a theory that he maintains is "frankly dynamic," with the theory of Dedekind, which he refers to as "static." Nowhere in Dedekind's definition of real number, says Dantzig, does Dedekind even "use the word *infinite* explicitly, or such words as *tend, grow, beyond measure, converge, limit, less than any assignable quantity,* or other substitutes."

At this point, reading Dantzig's account, we seem to have come to a resting place, for Dantzig writes:

> So it seems at first glance that here [in Dedekind's formulation of real numbers] we have finally achieved a complete emancipation of the number concept from the yoke of time.

To be sure, this "complete emancipation" hardly holds up to Dantzig's second glance, and the eternal issues regarding time and its mathematical representation, regarding the continuum and its relationship to physical time, or to our lived time---problems we have been made aware of since Zeno---remain constant companions to the account of the evolution of number you will read in this book.

Dantzig ends part I "The evolution of the number concept" with a discussion of the question: to what extent does the world, the scientific world, enter crucially as an influence on the mathematical world, and vice versa?

> The man of science will act *as if* this world were an absolute whole controlled by

laws independent of his own thoughts or act; but whenever he discovers a law of striking simplicity or one of sweeping universality or one which points to a perfect harmony in the cosmos, he will be wise to wonder what role his mind has played in the discovery, and whether the beautiful image he sees in the pool of eternity reveals the nature of this eternity, or is but the reflection of his own mind.

Dantzig writes:

The mathematician may be compared to a designer of garments, who is utterly oblivious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago; to this day a shape will occasionally appear which will fit into the garment as if the garment had been made for it. Then there is no end of surprise and of delight!

This bears some resemblance in tone to the famous essay of the physicist Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," but
Dantzig goes on, by offering us his highly personal notions of *subjective reality* and *objective reality*. Objective reality, according to Dantzig, is an impressively large receptacle including all the data that humanity has acquired, e.g., through the application of scientific instruments. He adopts Poincaré's definition of objective reality, "what is common to many thinking beings and could be common to all,"
to set the stage for his analysis of the relationship between number and objective truth.

Now, in at least one of Immanuel Kant's reconfigurations of those two mighty words *subject* and *object,* a dominating role is played by Kant's delicate concept of the *sensus communis*. This *sensus communis* is an inner "general voice," somehow constructed within each of us, that gives us our expectations of how the rest of humanity will judge things.

The *objective reality* of Poincaré and Dantzig seems to require, similarly, a kind of inner voice, a faculty residing in us, telling us something about the rest of humanity: The Poincaré -Dantzig *objective* reality is a fundamentally *subjective* consensus of what is commonly held, or what *could be* held, to be objective. This view already alerts us to an underlying circularity lurking behind many discussions regarding objectivity and number, and, in particular behind the sentiments of the essay of Wigner. Dantzig treds lightly around this circularity, perhaps heeding Robert Graves' "Warning to Children":

Children, leave the string alone!
For who dares undo the parcel
Finds himself at once inside it.

My brother Joe and I gave our father, Abe, a copy of "Number: The Language of Science" as a gift, when he was in his early 70s. Abe had no mathematical education beyond high school, but retained an ardent love for the algebra he learned there. Once, when we were quite young, Abe imparted some of the marvels of algebra to us: "I'll tell you a secret," he began, in a conspiratorial voice. He proceeded to tell us how, making use of the magic power of the cipher X we could find *that number which when you double it and add one to it you get 11*. I was quite a literal-minded kid and really thought of X as our family's secret, until I was disabused of this attribution in some math class a few years later.

Our gift of Dantzig's book to Abe was an astounding hit. He worked through it, blackening the margins with notes, computations, exegeses; he read it over and over again. He engaged with numbers in the spirit of this book; he tested his own variants of the Goldbach Conjecture and called them his *Goldbach Variations*. He was, in a word, enraptured.

But none of this is surprising, for Dantzig's book captures both soul and intellect; it is one of the few great popular expository classics of mathematics truly accessible to everyone.