

Thinking about Grothendieck

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During the early 60's his conversations had a secure calmness. He would offer mathematical ideas with a smile that always had an expanse of generosity in it. Firm feet on the ground; sometimes barefoot. Transparency: his feelings towards people, towards things, were straightforwardly felt, straightforwardly expressed—often garnished with a sprig of morality. But perhaps the word 'morality' doesn't set the right tone: one expects a dour or dire music to accompany any moral message. Grothendieck's opinions, observations, would be delivered with an upbeat, an optimism, a sense that "nothing could be easier in the world" than to view things as he did. In fact, as many people have mentioned, Grothendieck didn't butt against obstacles, but rather he arranged for obstacles to be dissolved even before he approached them. The mathematical road, he would seem to say, shows itself to be 'the correct way' by how easy it is to travel along it. This is, of course, a vastly different 'ease' than what was an intellectual abomination to Grothendieck: something he called, with horror, "tourner la manivelle" (or 'cranking it out').

Simplicity was a great virtue for him, in ideas, in material possessions, in food. The main objects in his living-room when he lived in an apartment at Résidence Gratien, in Bures-sur-Yvette, were a wrought-iron statue of a goat, a large urn filled with oil-cured black olives, a small somewhat rickety table on which perched his typewriter (his work-space). You could meet him on the way from market, during the weekly market-day in Bures, carrying only one (ample) bag of grapes, eating them as he walked and offering them to you.

His hospitality was startling. Later, when he lived near the RER stop Massy-Verrière he once invited an entire family who needed lodging, to stay in his basement and to bring with them their in-laws. He helped them install a taramasalata machine there to give them some economic activity.

In encountering a shopping mall when he visited Cambridge (USA) his only utterance was an Elizabethan "Let us flee." How sparing he was in any activity other than mathematics during the sixties. As a result, some of his non-mathematical experiences at that time had revelatory force for him. He returned to Bures from Paris one day (this was probably the late sixties) saying that he'd just seen the first movie he had seen in 12 years (Butch Cassidy and the Sundance Kid) and was struck by its moral complexity. The one non-mathematical book I know he was reading with intense respect at that epoch was a volume entitled "History of the Jews" (I've forgotten its author, or the language in which Grothendieck was reading it). John Tate writes that *Moby Dick* was Grothendieck's favorite novel.

After Grothendieck officially left the IHES, he did show up at the IHES a few times. Gretchen and I would invite him for lunch in the pavilion we were staying at in the Résidence de l'Ormaille. The predominant theme and message of his conversation at these lunches was how much he felt mathematics to be a siren-song: a distraction, and how I should free myself from it to open up to a wider psychologically-aware existence. (I would give counter-arguments.)

The mathematical talks I had with him—as I remember them now—were largely, perhaps only, about viewpoint, never about specifics (with the exception of a conversation about differential structures on conjugate complexifications of an algebraic variety over a number field). Grothendieck's message was clear throughout: that everything important will follow easily, will flow, from the right vantage. It was principally 'the right vantage,' a way of seeing mathematics, that he sought, and perhaps only on a lesser level, its by-products.

People have written about Grothendieck's intense category-theoretic genius. The phrase 'category-theoretic,' as far as it goes, is correct as a very vague pointer to Grothendieck's attitude, where, for example, Yoneda's Lemma plays such an important role. Yoneda's Lemma asserts that an object X of a category is determined (up to unique isomorphism) by the functor that records morphisms from X to *each* of the objects of that category. Or, in more evocative terms, a mathematical object X is best thought of in the context of a category surrounding it, and is determined by the network of relations it enjoys with *all* the objects of that category. Moreover, to understand X it might be more germane to deal directly with the functor representing it. This is reminiscent of Wittgenstein's 'language game'; i.e., that the meaning of a word is—in essence—determined by, in fact is nothing more than, its relations to all the utterances in a language.

Treating objects as functors was second nature to him, but that was the least of it: Grothendieck's view goes much further than that. For example—as if it were the most natural thing in the world—the mathematical objects X that he dealt with would often be defined directly in the context of all possible families of variations X_t of those objects (e.g., S -schemes, rather than algebraic varieties). On top of this, one of the (vast number of) great innovations he is responsible for gives us a deep understanding of what it means to pass from a global view of an object to a more local view of 'locales' in the object, or—going the other way—to agglomerate from the local to the global.

These words, of course, hardly begin to touch on the grandeur of the person he was, or of the ideas he has taught us.