

Notes on Utility

B. Mazur

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About our seminar-course “Utility”

Two years ago I had the pleasure of teaching a seminar on *Models* with my friends Eric Maskin and Amartya Sen. We covered mathematical models of all shapes and purposes; models in aid of statistics, or used in economics and in the theory of social choice. We dealt with models designed for a wide assortment of specific structures—auctions, voting procedures, for example.

In this up-coming seminar-course ‘Utility,’ (PHIL 273A) I look forward to as enjoyable an examination of the concept ‘Utility’ interpreted generously to include ideas about ‘usefulness,’ ‘purpose,’ and broad human desires (e.g., *happiness, justice*), as well as specific problems related to ‘strategy’ and ‘assessment of value’ in connection with the pursuit of particular goals.

‘Measurement of utility’, alone, has a wide-ranging literature related to

- behavioral issues (e.g., as developed by Kahnemann and Tversky),
- mathematical issues (e.g., the—often impossible—task of optimizing two or more competing preferences at the same time—leading to its formulation in terms of Game Theory as done by von Neumann and Morgenstern), and
- issues central to Economics and the Theory of Social Choice (e.g., ‘utility functions’ have played an interesting role in shaping the format of models in those fields).

Here are five specific aspects that we will explore:

1. Personal questions of ethics and meaning:

- Greek origins of utility as in Aristotle’s trichotomy of utility, pleasure, and virtue as the three goads for friendship ([5] *Nichomachean Ethics* Book VIII) from which follows a broad discussion of the nature of friendship,
- or Epicurus’s linking of utility to happiness ([8]) and his discussion of justice: *Natural justice is a symbol or expression of usefulness, to prevent one person from harming or being harmed by another*— leading to his definition of justice: *useful for mutual association*—

- and continuing to Kant’s dictum of treating people as *ends* in themselves rather than as useful *means* to some other end. ([16]).

2. Social moral issues related to utilitarian ideas:

Pro:

- **Hume.** Although there is no suggestion of a strategy for, or even a taste for, ‘maximization’ of utility in Hume’s writings, the concept itself plays an exceedingly important role in his thought. See Section V (*Why Utility pleases*) in the Enquiry concerning the Principles of Morals [14]. E.g.:

It seems so natural a thought to ascribe to their utility the praise, which we bestow on the social virtues, that one would expect to meet with this principle every where in moral writers, as the chief foundation of their reasoning and enquiry. In common life, we may observe, that the circumstance of utility is always appealed to; nor is it supposed, that a greater eulogy can be given to any man, than to display his usefulness to the public, and enumerate the services, which he has performed to mankind and society. What praise, even of an inanimate form, if the regularity and elegance of its parts destroy not its fitness for any useful purpose! And how satisfactory an apology for any disproportion or seeming deformity, if we can show the necessity of that particular construction for the use intended! A ship appears more beautiful to an artist, or one moderately skilled in navigation, where its prow is wide and swelling beyond its poop, than if it were framed with a precise geometrical regularity, in contradiction to all the laws of mechanics. A building, whose doors and windows were exact squares, would hurt the eye by that very proportion; as ill adapted to the figure of a human creature, for whose service the fabric was intended. What wonder then, that a man, whose habits and conduct are hurtful to society, and dangerous or pernicious to every one who has an intercourse with him, should, on that account, be an object of disapprobation, and communicate to every spectator the strongest sentiment of disgust and hatred.

- **J.S. Mill.** ([20]) His treatise *Utilitarianism* on the one hand, offers general guidelines (and formats) for rational argument about moral principles in general, and on the other, describes and defends utilitarianism as a viewpoint framed in broad terms to incorporate human desire and questions of happiness.

The utilitarian doctrine is that happiness is desirable as an end, and is the only thing that is so; anything else that is desirable is only desirable as means to that end.

- **Jeremy Bentham** and **J.S. Mill.** I found sections 2.1 and 2.2 of [14], the entry *Utilitarianism* in the *Stanford History of Philosophy* extremely useful for a discussion of the interplay of ideas (of Bentham and Mill) regarding utilitarianism.

Anti:

- **Kant** ([16]) Regarding the principle of “greatest happiness for the greatest number,” Kant is not wishy-washy:

... it is odd how it could have occurred to intelligent men, [merely] because the desire for happiness and hence also the maxim whereby everyone posits this happiness as the determining basis of his will is universal, to therefore pass this [maxim] off as a universal practical law. For although ordinarily a universal law of nature makes everything accordant, here, if one wanted to give to the maxim the universality of a law, precisely the extreme opposite of accordance would result: the gravest conflict, and the utter annihilation of the maxim itself and of its aim. For then the will of all does not have one and the same object, but each person has his [own] object (viz., his own well-being); and although contingently this object may indeed be compatible with the aims of other people as well, who likewise direct them at themselves, it is far from being sufficient for a law, because the exceptions that one is occasionally authorized to make are endless and cannot at all be encompassed determinately in a universal rule. In this way there results a harmony similar to that depicted by a certain satirical poem on the concord of soul between a married couple who are [bent on] bringing themselves to ruin: "O marvelous harmony, what he wants she also wants," etc.; or to what is reported about the pledge made by King Francis I against Emperor Charles V: "What my brother Charles wants to have (Milan) I also want to have."

Empirical determining bases are not suitable for any universal external legislation, but just as little also for an internal one; for each person lays at the basis of inclination his [own] subject, but another person another subject; and in each subject himself now this inclination and now another is superior in influence. Discovering a law that under this condition would govern them all [viz., with accordance on all sides] is absolutely impossible.

- **Smith** ([10] IV.2.4, IV 2.9): Here, Smith offers the eponymous example of a general species of reasoning that Nozick [21] nicely labeled *invisible hand arguments*:

Every individual is continually exerting himself to find out the most advantageous employment for whatever capital he can command. It is his own advantage, indeed, and not that of the society, which he has in view. But the study of his own advantage naturally, or rather necessarily, leads him to prefer that employment which is most advantageous to the society...he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it.

- **Nozick.** (see especially pp. 28-30 in [21]). Nozick works out a (libertarian) defense of something like a 'minimal state.' This may not be exactly the 'Nightwatchman state,' which restricts its activities to bodily protection of its citizens and the enforcement of contracts, but it is close to that. In such a state, looking out for the 'happiness' of its citizens is not one of its primary goals—although, he allows—it might be a consequence. Rather, Nozick wishes to have states whose primary goals are the ('maximalization of the') nonviolation of individual's rights. He toys with the phrase 'utilitarian of rights.'
- **Rawls.** Rawls's objection to utilitarianism also has to do with its relationship to individual rights. He wants (see (xi) of [22]):

... to work out a conception of justice that provides a reasonably systematic alternative to utilitarianism, which in one form or another has long dominated the Anglo-Saxon tradition of political thought. The primary reason for wanting to find such an alternative is the weakness, so I think, of utilitarian doctrine as a basis for the institutions of constitutional democracy. In particular, I do not believe that utilitarianism can provide a satisfactory account of the basic rights and liberties of citizens as free and equal persons, a requirement of absolutely first importance for an account of democratic institutions.

He writes that the first objective of his take on justice (*justice as fairness*) is to emphasize the priority of basic rights and liberties. He “used a more general and abstract rendering of the idea of the social contract by means of the idea of the original position as a way to do that.”

3. On the usefulness of ‘theory’.

- Here is G.H. Hardy, a mathematician of the early twentieth century discussing this in his *A Mathematician’s Apology*:

Is mathematics ‘useful, directly useful, as other sciences such as chemistry and physiology are? This is not an altogether easy or uncontroversial question, and I shall ultimately say No, though some mathematicians, and some outsiders, would no doubt say Yes. And is mathematics ‘harmless? Again the answer is not obvious, and the question is one which I should have in some ways preferred to avoid, since it raises the whole problem of the effect of science on war. Is mathematics harmless, in the sense in which, for example, chemistry plainly is not?

The publication date of *A Mathematician’s Apology*, 1940, is relevant for an appreciation of what underlies these sentiments.

- And here is an excerpt of Abraham Flexner’s eloquent essay *The Usefulness of Useless Knowledge*. (Flexner was the founding director of the Institute of Advanced Study at Princeton.)

From a practical point of view, intellectual and spiritual life is, on the surface, a useless form of activity, in which men indulge because they procure for themselves greater satisfactions than are otherwise obtainable. In this paper I shall concern myself with the question of the extent to which the pursuit of these useless satisfactions proves unexpectedly the source from which undreamed-of utility is derived.

The publication date, 1939, is also significant.

- And here is Adam Smith’s curious comment on the relationship between usefulness and beauty in the sciences (*Theory of Moral Sentiments*¹).

It is in the abstruser sciences, particularly in the higher parts of mathematics, that the greatest and most admired exertions of human reason have been

¹This is from [9] Chapter II *Of the beauty which the appearance of Utility bestows upon the characters and actions of men...* in Part IV *Of the Effect of Utility upon the Sentiment of Approbation*. It was published in 1759.

displayed. But the utility of those sciences, either to the individual or to the public, is not very obvious, and to prove it, requires a discussion which is not always very easily comprehended. It was not, therefore, their utility which first recommended them to the public admiration. This quality was but little insisted upon, till it became necessary to make some reply to the reproaches of those, who, having themselves no taste for such sublime discoveries, endeavoured to depreciate them as useless.

4. **Aesthetic aspects:** As in Kant’s discussion of *purposiveness*² as opposed to *purpose* and *usefulness* in *The Analytic of the Beautiful* ([17]; Introduction pp. 20-28; Book I Sections 10, 11 pp. 64-67).

... we do call objects, states of mind, or acts purposive even if their possibility does not necessarily presuppose the presentation of a purpose; we do this merely because we can explain and grasp them only if we assume that they are based on a causality [that operates] according to purposes, i.e., on a will that would have so arranged them in accordance with the presentation of a certain rule. Hence there can be purposiveness without a purpose, insofar as we do not posit the causes of this form in a will, and yet can grasp the explanation of its possibility only by deriving it from a will. Now what we observe we do not always need to have insight into by reason (as to how it is possible). Hence we can at least observe a purposiveness as to form and take note of it in objects even if only by reflection-without basing it on a purpose.³

or the ‘form follows function’ dictum in modern architecture.

5. Formal and more mathematical tools.

- *Initial ideas of utility and utility-maximizing.*
Daniel Bernoulli (1738) ([7]⁴)
- *‘Betting behavior’ as a way of formalizing personal preferences of utility.*
Frank Ramsey, Bruno de Finetti; choice-based subjective probability: see [23]. The use of utility functions as representation of preferences.
- *Arguments against the use of a determinate utility function*
Lionel Robbins [24]:

Bailey pointed out over a hundred years ago, “As we cannot speak of the distance of any object without implying some other object between which and the former this relation exists, so we cannot speak of the value of a commodity, but in reference to another commodity compared with it. A thing cannot be valuable in itself without reference to another thing, any more than a thing can be distant in itself without reference to another thing.”

It follows from this that the term which, for the sake of continuity and to raise certain definite associations, we have used hitherto in this chapter, the term “economic quantity” is really very misleading. A price, it is true, expresses the

²Zweckmässigkeit

³All this hangs a bit on Kant’s rather curious notion of *the will* (cf. loc.cit.)

⁴And for modern discussions regarding the St. Petersburg Paradox, one of the focusses of Bernoulli’s thought, see [11], [12].

quantity of money which it is necessary to give in exchange for a given commodity. But its significance is the relationship between this quantity of money and other similar quantities. And the valuations which the price system expresses are not quantities at all. They are *arrangements in a certain order*. To assume that the scale of relative prices measures any quantity at all save quantities of money is quite unnecessary. Value is a relation, not a measurement... Recognition of the ordinal nature of the valuations implied in price is fundamental. It is difficult to overstress its importance. With one slash of Occam's razor, it extrudes for ever from economic analysis the last vestiges of psychological hedonism.

- *Game Theoretical format.*

von Neumann-Morgenstern. The starting-point of their classic text is

to obtain a real understanding of the problem of exchange by studying it from an altogether different angle; this is, from the perspective of a “game of strategy”⁵

leading the authors to consider models of social exchange economy that represent individuals exposed to a constellation of social influences exposed to multiple factors. They formalize this by stipulating, in their model, that each party attempts to maximize his ‘interests’ given that he does not control all variables.

This is certainly no maximum problem, but a peculiar and disconcerting mixture of several conflicting maximum problems. Every participant is guided by another principle and neither determines all variables which affect his interest.

See [27] and read especially pp. 1-48 as given in the on-line link in [27]. For a somewhat less technical text covering the mathematics involved, cf. [18].

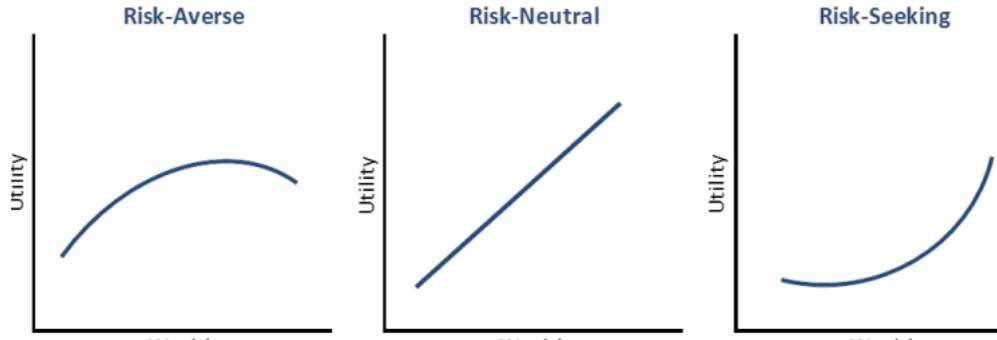
- *Axiomatic preference relations interpreted as the maximization of the expectations of a utility function on the set of consequences with respect to a probability measure on the set of all events.*

L.J. Savage (1954) [26]. Also: **Gerard Debreu** (1954) and **Ken Arrow** (1972); see [6].

6. 'Behavioral' issues:

- ‘Measurement of happiness.’
- The utility of money.
- Interpersonal comparisons—including the informational basis of comparability connected to things discussed above.
- Models to describe a participant's re-evaluation of preferences related to marketplace issues—given changes of wealth—the nature of this re-evaluation depending on overall psychological temperament—ranging from the risk-averse to the thrill-seeking.

⁵This is in contrast to earlier “Robinson Crusoe” models that involve only a single actor balancing his various preferences to make his next move.



- 'Behavioral challenges' to expected utility:

- See Lionel Robbins [25]:

My own attitude to problems of political action has always been one of what I might call provisional utilitarianism... But, as time went on, things occurred which began to shake my belief in the existence of so complete a continuity between politics and economic analysis. I began to feel that there were profound difficulties in a complete fusion between ... the economic and the hedonistic calculus. It did not take long to see that the "law" of diminishing marginal utility, assumed ... in the analysis of inequality, differed from the "law" of the same name invoked in the analysis of exchange and that the difference was precisely the introduction of [the assumption that all humans] have equal capacity for satisfaction.

- The 'Allais Paradox,' [19]; see also the exposition in the Wikipedia entry [2].
- The 'Ellsberg Paradox'[8]; see also the exposition in the Wikipedia entry [3].
- Kahneman-Tversky's 'Prospect Theory' [15].

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For pp. 1-48 of this text, see:
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