(1) Let \( f : R \to R' \) be a flat map of commutative rings, and suppose that \( f \) induces a surjection \( F : \text{Spec } R' \to \text{Spec } R \). Show that \( F \) is a quotient map: that is, a subset \( U \subseteq \text{Spec } R \) is open if and only if \( F^{-1}U \) is an open subset of \( \text{Spec } R' \).

(2) Let \( R \) be a local Noetherian ring, and let \( R^\vee \) denote the completion of \( R \) (with respect to its maximal ideal). Show that the depth of \( R \) coincides with the depth of \( R^\vee \).

(3) Let \( R \) be a local Noetherian ring. Show that if \( R \) is Cohen-Macaulay, then there exists a (nonzero) finitely generated \( R \)-module \( M \) whose length and projective dimension are both finite.

(4) Let \( R \) be a local Noetherian ring which contains the field \( \mathbb{Q} \) of rational numbers, let \( G \) be a finite group acting on \( R \), and let \( R^G \subseteq R \) be the fixed points for the action of \( G \). Show that \( R^G \) is also a local Noetherian ring, which is Cohen-Macaulay if \( R \) is Cohen-Macaulay.