(1) Show that the map \( f : \mathbb{C} - \{0\} \to \mathbb{C} - \{0\} \), given by \( f(z) = z^n \), is a covering map.

(2) Let \( S^1 \) denote the unit circle \( \{z \in \mathbb{C} : |z| = 1\} \), and let \( B^2 \) denote the unit ball \( \{z \in \mathbb{C} : |z| \leq 1\} \). Let \( X \) be a topological space. Prove that the following conditions are equivalent:

(a) For every point \( x \in X \), the fundamental group \( \pi_1(X, x) \) is trivial.

(b) Every continuous function \( f_0 : S^1 \to X \), there exists a continuous map \( f : B^2 \to X \) which extends \( f_0 \).

Suppose that \( X, Y, \) and \( Z \) are topological spaces, and that we are given continuous maps \( f : X \to Z \) and \( g : Y \to Z \). The fiber product \( X \times_Z Y \) is the set \( \{(x, y) \in X \times Y : f(x) = g(y)\} \subseteq X \times Y \). We regard \( X \times_Z Y \) as a topological space by equipping it with the subspace topology.

(3) Let \( f : \tilde{X} \to X \) be a covering map and let \( g : X' \to X \) be arbitrary continuous function. Show that the projection map \( \tilde{X} \times_X X' \to X' \) is a covering map.

(4) A continuous map of topological spaces \( f : X \to Y \) is said to be proper if, for every continuous map \( Y' \to Y \), the projection map \( X \times_Y Y' \to Y' \) is closed. Show that if \( f \) is closed and the fiber \( f^{-1}\{y\} \) is compact for each \( y \in Y \), then \( f \) is proper.

(5) Show that a topological space \( K \) is compact if and only if, for every topological space \( Z \), the projection map \( K \times Z \to Z \) is closed. Use this to prove the converse to (4): if \( f : X \to Y \) is a proper map of topological spaces, then \( f \) is closed and the fiber \( f^{-1}\{y\} \) is compact for each \( y \in Y \). (Hint: let \( A \) be a filtered partially ordered set and let \( A' = A \cup \{\infty\} \). Show that there is a topology on \( A' \) such that if the projection map \( K \times A' \to A' \) is closed, then every net \( A \to K \) has an accumulation point.)