Syllabus for Math 277: Combinatorial representation theory

- **Lectures**: Tuesday and Thursday 1pm – 2:30pm, in Science Center 411.
- **Lecturer**: Lauren Williams
  - **Course webpage**: http://www.math.harvard.edu/~ lauren/CRT.html
  - **Office**: Science Center 432
  - **Email**: lauren@math.harvard.edu
  - **Office Hours**: TUESDAYS 2:30pm-3:30pm (tentative) and by appointment
- **Course Assistant**: ??
- **Course Description**: A lot of interesting combinatorics is a "shadow" of phenomena in representation theory. We will see this as we learn about Kashiwara's crystal bases and Fomin and Zelevinsky's cluster algebras, and related combinatorial objects such as tableaux, generalized associahedra, etc. Although crystal bases and cluster algebras are different subjects, there is a connection between them, via Lusztig's canonical basis. One gets crystal bases by taking the limit of the canonical basis as q goes to 0. And cluster algebras are related to the dual canonical basis (this statement is somewhat conjectural).
- **Prerequisites**: You should either be familiar with root systems, semi-simple Lie algebras, and their representations, or be willing to learn very quickly. I plan to do a quick review in one or two lectures of this material, but it honestly doesn't make much sense for you to take this class if you haven't seen some Lie theory already.
- **Textbooks**: Hong and Kang "Introduction to quantum groups and crystal bases." There is not book on cluster algebras so I'll draw from papers of Fomin and Zelevinsky.
- **Grading**: There will be some problem sets for those people who need a grade. Also, those students taking the class for a grade will be expected to give a 40-minute presentation later in the course.
- **Syllabus**: I plan to cover the following topics.
  - Definition of crystal bases
  - The tensor product rule for crystal bases
  - Littelmann's path model for constructing crystals
  - Other realizations of crystals in terms of tableaux, etc
  - Stembridge's local characterization of simply-laced crystals
  - Definition of cluster algebras
  - Finite type classification of cluster algebras
  - Cluster algebra combinatorics, including generalized associahedra
  - Examples from geometry, eg Grassmannians, double Bruhat cells, triangulated surfaces
  - Y-systems and more generally the dynamics of coefficients in cluster algebras