

**MATH 277 PROBLEM SET 3**  
**(DUE TUESDAY, DECEMBER 2)**

Do all of the below problems.

- (1) Use the caterpillar lemma to prove that for any positive integers  $a, b, c$ , if the sequence  $y_0, y_1, \dots$  satisfies the recurrence

$$y_k = \frac{y_{k-3}^a y_{k-1}^c + y_{k-2}^b}{y_{k-4}},$$

then each  $y_k$  is a Laurent polynomial in the variables  $y_0, y_1, y_2, y_3$ .

- (2) Consider the cluster algebra associated to the B-matrix (with all entries 0 or  $\pm 1$ ) whose adjacency graph is represented in Figure 1. Either show that this cluster algebra is of finite type (and identify what type it is), or show that it is not of finite type.

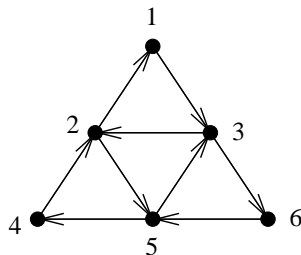


FIGURE 1. The quiver of a B-matrix

- (3) Use the tropical semifield, and start with the seed  $\Sigma_0 = ((x_1, x_2), (y_1, y_2), B_0)$  where  $B_0$  is equal to

$$\begin{pmatrix} 0 & 1 \\ -3 & 0 \end{pmatrix}.$$

Draw the exchange graph for this cluster algebra together with all seeds. Of what type is this cluster algebra?

(4) Consider the cluster algebra associated to the  $B$ -matrix given by

$$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Either show that this cluster algebra is of finite type (and identify what type it is), or show that it is not of finite type.