

MATH 277 PROBLEM SET 2
(DUE THURSDAY, NOVEMBER 6)

Do all of the below problems.

- (1) Use Littelmann paths to find the crystal $B(\epsilon_1)$ of the type B_2 highest weight representation with highest weight ϵ_1 .
- (2) Use Stembridge's algorithm to find the crystal $B(\epsilon_1)$ of the type D_4 representation with highest weight ϵ_1 .
- (3) For type D_4 again, take the tensor product $B(\epsilon_1) \otimes B(\epsilon_1)$ in order to find the crystal $B(2\epsilon_1)$.
- (4) Draw the \mathfrak{sl}_3 crystal $B(\lambda)$ in terms of semistandard tableaux for $\lambda = (3, 2, 1)$.
- (5) Use the Littlewood Richardson Rule for \mathfrak{sl}_n we saw in class (Lecture 6) to express how $B((2, 1)) \otimes B((3, 2, 1))$ decomposes into irreducible (connected) crystals.