PROFINITE PROPERTIES OF GRAPH MANIFOLDS

HENRY WILTON AND PAVEL ZALESSKII, REVIEWED BY THOMAS KOBERDA

The authors prove several separability theorems for a large class of 3–manifolds. The authors start with a compact, orientable, irreducible 3–manifold M. Such a manifold admits a JSJ–decomposition, which is to say a canonical finite collection of embedded, disjoint tori $T_1, \dots, T_k$ such that

$$M \setminus \bigcup_{i=1}^{k} T_i$$

consists of a finite collection of atoroidal manifolds and Seifert fibered manifolds. If the boundary of M consisted of tori then each resulting component of

$$M \setminus \bigcup_{i=1}^{k} T_i$$

admits a finite volume geometric structure. The authors give the hypothesis that each piece of M be geometrizable, a hypothesis which is implied by the resolution of the Geometrization Conjecture by the work of Perelman.

The JSJ–decomposition of M gives $G = \pi_1(M)$ the structure of a graph of groups. The profinite topology on G is said to be efficient if each vertex and edge group in the graph of groups decomposition of G is closed. The first theorem in the article is that the profinite topology on G is efficient.

A group G is called good if the natural map from G to its profinite completion induces an isomorphism on cohomology with coefficients in each finite G–module. The authors then show that if each geometric piece of M has a good fundamental group then so does M. Many geometric manifolds have good fundamental groups, including arithmetic hyperbolic manifolds and Seifert fibered manifolds. In particular, graph manifolds have good fundamental groups.

A group is called conjugacy separable if each conjugacy class is closed in the profinite topology. The final theorem of the article is that a graph manifold has a conjugacy separable fundamental group.

The proofs proceed by a combination of methods, including actions of groups on trees, certain separability results in Haken manifolds following E. Hamilton, and work of Serre on good groups.

Department of Mathematics, Harvard University, 1 Oxford St., Cambridge, MA 02138

E-mail address: koberda@math.harvard.edu