

ON THE FOURTH JOHNSON HOMOMORPHISM OF THE AUTOMORPHISM GROUP OF A FREE GROUP

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Let G be a group and let $\text{Aut}(G)$ denote its automorphism group. Recall the lower central series $\{\gamma_i(G)\}$ is defined by $\gamma_0(G) = G$ and $\gamma_{i+1}(G) = [G, \gamma_i(G)]$. We have that $\text{Aut}(G)$ acts on $G_i = G/\gamma_i(G)$. The kernel J_i of this action is a subgroup of $\text{Aut}(G)$. When G is a free group or a surface group, the central filtration $\{J_i\}$ is often called the Johnson filtration, after the work of D. Johnson [*Low-dimensional topology (San Francisco, Calif., 1981)*, 165–179, *Contemp. Math.*, 20, *Amer. Math. Soc., Providence, RI*, 1983; MR0718141 (85d:57009)].

Each graded piece $\mathcal{L}_i(G) = \gamma_i(G)/\gamma_{i+1}(G)$ of the lower central series of G can be viewed as a quotient of $H^1(G) \otimes_{\mathbb{Z}} \mathcal{L}_{i-1}(G)$, and we in fact get equality when G is a free group. The n^{th} Johnson homomorphism is a map from the n^{th} graded piece of the Johnson filtration to

$$H^1(G) \otimes \mathcal{L}_{n+1}(G),$$

given by the Lie bracket.

In the present article, the author computes with rational cokernel of the fourth Johnson homomorphism for the free group F_n and describes it as a module over $\text{GL}_n(\mathbb{Z})$. As a corollary, one can give a lower bound on the rank of the fourth graded piece of the Johnson filtration. The proofs proceed by explicit computation.

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