REAL ELEMENTS IN THE MAPPING CLASS GROUP OF T^2

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The author classifies the elements of the mapping class group of the torus T^2 which can be written as a composition of two orientation reversing involutions. It is quite easy to show that all finite order elements and parabolic elements in the mapping class group can be written as products of two orientation reversing involutions, and the author does this explicitly.

For hyperbolic mapping classes, the picture is somewhat more complicated. The author associates a so–called cutting period–cycle to the hyperbolic geodesic stabilized by each hyperbolic matrix. Roughly this is the periodic sequence of left and right turns which must be made to follow the geodesic with respect to the generating matrices

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

The result is that a hyperbolic matrix is a product of two orientation reversing involutions if and only if the cutting period–cycle is odd–bipalindromic, which is to say it can be subdivided into two palindromic sequences of odd length.

The result fits in nicely with existing theory concerning involutions and other torsion elements in mapping class groups. The motivation for this result comes from the author's study of Lefschetz fibrations.

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