CANNON–THURSTON MAPS AND BOUNDED GEOMETRY

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In the present article, the author studies Cannon–Thurston maps for surface subgroups of hyperbolic 3–manifold groups. Let X and Y be Gromov hyperbolic metric spaces, and suppose $i : Y \to X$ is an embedding. A Cannon–Thurston map for i is a continuous extension of i from the boundary of Y to the compactification of X by its boundary.

After developing some general theory of hyperbolic metric spaces with a view on Cannon–Thurston maps, the author gives proofs of the following results:

(1) (Cannon–Thurston): Let M be a hyperbolic 3–manifold which fibers over the circle with fiber F. Then there exists a Cannon–Thurston map $\mathbb{H}^2 \cup S^1 \to \mathbb{H}^3 \cup S^2$ which extends the inclusion of the universal cover of F into the universal cover of M.

(2) (Minsky): Let Γ be a Kleinian surface group such that $\mathbb{H}^3/\Gamma$ has bounded geometry, which is to say its injectivity radius is bounded from below. Then there exists a continuous map from the Gromov boundary of Γ to the limit set of Γ in the boundary of $\mathbb{H}^3$.

(3) (Bowditch): Let S be a hyperbolic surface of finite volume and let N be a hyperbolic manifold corresponding to a representation of $\pi_1(S)$ without accidental parabolics. Let $i : S \to N$ be a proper homotopy equivalence. Then the lift of i to the universal cover of S and N extends to the respective boundaries.

The proofs given in the article are detailed, making this a readable, self–contained exposition.

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