GROTHENDIECK'S PROBLEM FOR 3–MANIFOLD GROUPS

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Grothendieck's problem asks the following: if $H < G$ is a finitely presented subgroup of a finitely presented residually finite group and the induced map on profinite completions is an isomorphism, does it follow that the inclusion map was an isomorphism? The general case was solved negatively by Bridson and Grunewald [Ann. of Math. (2) 160 (2004), no. 1, 359–373; MR2119723 (2005k:20069)]. A pair (H, G) with $H < G$ is a Grothendieck pair if it witnesses a counterexample to Grothendieck's problem. If for all finitely generated subgroups H of G the pair (H, G) is never a Grothendieck pair, we call G Grothendieck rigid. The primary results of the paper are that closed geometric 3–manifolds and finite volume hyperbolic 3–manifolds have Grothendieck rigid fundamental groups. Outside of the realm of geometry, the authors show that a closed, irreducible rational homology sphere has a Grothendieck rigid fundamental group.

The authors pose a related question, namely if M_1 and M_2 are geometric manifolds with infinite fundamental groups whose profinite completions are isomorphic, does it follow that M_1 and M_2 are homeomorphic? The Grothendieck problem is related to subgroup separability, as the authors show that a LERF group is Grothendieck rigid.

The proofs of the results use methods from hyperbolic geometry and representation varieties.

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