PING–PONG AND OUTER SPACE

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In the present article, the authors set up ping–pong on the Thurston compactification of outer space for a free group. The theory is analogous to the case of surfaces and mapping class groups. The mapping class group of a surface acts on the space of projective measured laminations on the surface, and these form a natural compactification of Teichmüller space. Certain distinguished elements of the mapping class group known as pseudo-Anosov elements fix unique Teichmüller geodesics whose endpoints form a pair of dual laminations. If ψ and ϕ are two pseudo-Anosov elements which do not share a common power then they stabilize two distinct Teichmüller geodesics and the two pairs of endpoints are distinct. After passing to sufficiently large powers of ψ and ϕ, one can conclude that the resulting mapping classes generate a nonabelian free group. The argument used to prove that the resulting group is free is known as the ping–pong lemma, and is originally due to Klein [Neue Beiträge zur Riemann'schen Functionentheorie. (German) *Math. Ann.* 21 (1883), no. 2, 141–218; MR1510193]. By choosing ψ and ϕ and the powers carefully, Mosher showed that one can actually produce a nonabelian free subgroup of the mapping class group which consists entirely of pseudo-Anosov homeomorphisms [*Proc. Amer. Math. Soc.* 125 (1997), no. 12, 3447–3455; MR1443845 (98c:20069)].

The authors prove that certain analogues of pseudo-Anosov homeomorphisms, known as a hyperbolic irreducible automorphism with irreducible powers (hyperbolic iwip), share the above mentioned properties of pseudo-Anosov homeomorphisms. In particular, they show that two hyperbolic iwips which do not generate a virtually cyclic subgroup of the outer automorphism group of the free group generate a nonabelian free group after passing to sufficiently high powers. They also establish that a non–virtually cyclic subgroup of the outer automorphism group which contains a hyperbolic iwip contains a nonabelian free group in which each nonidentity element is a hyperbolic iwip.

The proofs of the main theorems are highly technical. The authors use the theory of geodesic currents on the free group and the description of outer space as a space of minimal, free, discrete and isometric free group actions on $\mathbb{R}$–trees.

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