HOMEWORK 6 SOLUTIONS

1. Problem 1

It is clear that X/A is Hausdorff, and the verification of all the continuity/finiteness properties of the characteristic maps is a matter of chasing definitions. b. This is obvious. c. Since (X, A) is a good pair, one can use the long exact sequence of a pair and excision to show that the homology of (X, A) is just the reduced homology of X/A.

2. Problem 2

a. Since -1 is a scalar in $\mathbb{R}$, multiplication by -1 commutes with each $\mathbb{R}$–linear map. b. No. Consider the canonical inclusion $V \to V \oplus \mathbb{R}$. c. No. The identity functor is covariant but the dual is contravariant.

3. Problem 3

a. Since abelian groups are $\mathbb{Z}$–modules, multiplication by n commutes with each homomorphism of abelian groups. b. This follows from the fact that T restricts to multiplication by n on any cyclic subgroup of any given abelian group. c. We already have that T is given by multiplication by n. Notice that multiplication by n on $\mathbb{Z}$ becomes multiplication by n^2 on $\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}$. Since T has to commute with all homomorphisms, it follows that T is the zero map.

4. Problem 4

The target category of each C_n has a natural addition given by direct sum, so that the natural transformations can be added in a natural way. Let $Nat(C_m, C_n) \to C_n\Delta[m]$ be given by

$$\eta \mapsto \eta_{\Delta[m]}(id_{\Delta[m]} : \Delta[m] \to \Delta[m]).$$

This is a homomorphism by construction. To construct an inverse, let $C_n\Delta[m] \to Nat(C_m, C_n)$ be given by taking $g : \Delta[n] \to \Delta[m]$ to the transformation $C_m \to C_n$ by the map induced by precomposition on the generators. One can show that this is an inverse and hence there is an isomorphism as claimed.