# SUBGROUPS GENERATED BY TWO PSEUDO-ANOSOV ELEMENTS IN A MAPPING CLASS GROUP. I. UNIFORM EXPONENTIAL GROWTH

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The author proves that under certain technical hypotheses, a sufficiently complicated group action by isometries on a $\delta$–hyperbolic graph implies the existence of a nonabelian free subgroup. The theory is then applied to the mapping class group $\mathrm{Mod}(S)$ acting on the curve complex of a non-sporadic surface $S$, which is $\delta$–hyperbolic by the work of Masur–Minsky [*Invent. Math.* 138 (1999), no. 1, 103–149; MR1714338 (2000i:57027)]. One of the main theorems proved as an application of the theory is that there is a constant $Q$ depending only on the surface such that if $a$ and $b$ are two pseudo-Anosov homeomorphisms which do not generate a virtually cyclic group, then there is an $M$ such that $a^n$ and $b^m$ or $a^m$ and $b^n$ generate a free group of rank two for all $n \geq Q$ and $m \geq M$. This is a strengthening of a well-known theorem about pseudo-Anosov homeomorphisms: if $a$ and $b$ are two pseudo-Anosov homeomorphisms which do not stabilize the same Teichmüller geodesic, then there are positive powers $n$ and $m$ such that $a^n$ and $b^m$ generate a free group [see Ivanov, *Subgroups of Teichmüller Modular Groups*, Translations of Mathematical Monographs, 115; MR1195787 (93k:57031)].

The methods used in the general theory are those of $\delta$–hyperbolic geometry of graphs. The main tool used is the notion of an acylindrical actions, following Bowditch [*Invent. Math.* 171 (2008), no. 2, 281–300; MR2367021 (2008m:57040)] to produce powers of hyperbolic isometries with large translation length.

The author closes with some remarks on uniform exponential growth in the mapping class group, recalling the work of Koubi [*Ann. Inst. Fourier* (Grenoble) 48 (1998), no. 5, 1441–1453; MR1662255 (99m:20080)] which shows that word–hyperbolic groups always have uniform exponential growth. He poses two questions, one concerning whether or not non-virtually abelian subgroups of the mapping class group have uniform exponential growth, and the other concerning the existence of uniformly short pseudo-Anosov elements in $\mathrm{Mod}(S)$. The former of these is answered positively by Mangahas [Uniform Uniform Exponential Growth of the Mapping Class Group, to appear in *Geom. Funct. Anal.*].

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