

# EXOTIC LEFT-ORDERINGS OF THE FREE GROUPS FROM THE DEHORNOY ORDERING

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Let  $G$  be a group and let  $<$  be a left ordering on  $G$ . The relation  $<$  is a total ordering on  $G$  which satisfies the following left-invariance property: whenever  $g, h, k \in G$  and  $g < h$  then  $k \cdot g < k \cdot h$ . A subgroup  $H$  of an ordered group  $(G, <)$  is called convex if whenever  $g, h \in H$  and  $k \in G$  satisfies  $g < k < h$ , we have  $k \in H$ .

The author uses a certain left ordering of the braid group called the Dehornoy ordering in order to exhibit orderings on finitely generated non-abelian free groups which admit no nontrivial, proper, convex subgroups. A corollary of the author's result is that the resulting ordered free groups are "exotic", in the sense that they are non-Conradian and are not bi-invariant.

An ordering is called Conradian if for any  $g, h > 1$ , there exists an  $n$  such that  $hg^n > g$ . A result of Conrad shows that whenever  $G$  admits a Conradian or a bi-invariant ordering which has no proper, nontrivial, convex subgroups is in fact a subgroup of  $(\mathbb{R}, +)$ .

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