PARASURFACE GROUPS

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It is a classical result of Magnus that if G is a finitely generated, residually nilpotent group whose lower central series is the same as that of a free group F_k of the same rank, in the sense that for each term γ_i of the lower central series we have

$$G/\gamma_i(G) \cong F_k/\gamma_i(F_k),$$

then $G \cong F_k$. Groups satisfying the hypotheses of G are known as **parafree**. Magnus' result can be stated succinctly as "parafree groups are free". In the present article, the author shows that an analogous theorem for surface groups cannot hold. The author defines a **parasurface group** to be a residually nilpotent group whose lower central series coincides with that of some closed surface group. The first main result of the paper is that there exist parasurface groups which are not isomorphic to any surface group.

The second main result of the paper is that free groups of even rank are not automorphism–nilpotent separable, a result which was already known for odd rank nonabelian free groups. Precisely, this means that there are two elements g and h of the free group which are not in the same automorphism orbit in the free group, but which are in the same automorphism orbit in each characteristic nilpotent quotient of the free group.

The final topic addressed in the paper concerns one–relator groups. It is a result of Magnus that if g and h are two elements of a free group whose normal closures are equal then g is conjugate to $h^{\pm 1}$. The author produces two words in the free group which, for each nilpotent quotient of the free group, have the same normal closure but are not conjugate in sufficiently complicated nilpotent quotients.

The proofs of the results are combinatorial and very explicit.

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