

SUBGROUPS OF PROFINITE SURFACE GROUPS

LIOR BARY-SOROKER, KATHERINE F. STEVENSON AND PAVEL A.
ZALESSKII, REVIEWED BY THOMAS KOBERDA

The authors study some aspects of the algebraic structure of the étale fundamental group of a projective curve over an algebraically closed field of characteristic zero. In particular, they study free and semi-free subgroups of profinite surface groups. It is easy to see that a naïve profinite version of the Nielsen–Schreier theorem fails, since there are non-free profinite subgroups of the profinite completion of \mathbb{Z} .

A group is called semi-free if every finite split embedding problem is properly solvable. The fundamental result of the article is as follows: let Π be a profinite surface group of genus two or more and let N be a closed subgroup of infinite index. Suppose there exist closed normal subgroups K_1 and K_2 whose intersection is contained in N but which themselves are not contained in N . Then N is semi-free of countable rank. A consequence of the main result is that if N is a closed normal subgroup of infinite index in Π then every proper open subgroup of N is semi-free.

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY, 1 OXFORD ST., CAMBRIDGE,
MA 02138

E-mail address: koberda@math.harvard.edu