

CONJUGACY SEPARABILITY OF SEIFERT 3-MANIFOLD GROUPS OVER NON-ORIENTABLE SURFACES

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The authors complete the theory of conjugacy separability of Seifert 3-manifold groups. Let $S \subset G$ be a subset of a group. Recall that G is S -separable if, for each $x \in G \setminus S$, there is a normal subgroup N_x of finite index such that $x \notin N_x S$. Equivalently, S is a closed subset of G in the profinite topology. A group is conjugacy separable if it is S_x separable for each $x \in G$, where S_x is the conjugacy class of x . Separability properties are related to solutions of certain decision problems in group theory, as well as lifting immersions to embeddings on regular covers of spaces.

Let G be a Seifert 3-manifold group, namely the fundamental group of a 3-manifold equipped with a certain decomposition into disjoint circles. This decomposition identifies a certain “base” surface. Seifert 3-manifold groups have known presentations (see for instance the book of Hempel [*AMS Chelsea Publishing*, Providence, RI, 2004. xii+195 pp.; MR2098385 (2005e:57053)]). Scott proves that G is subgroup separable [*J. London Math. Soc.* (2) 17 (1978), no. 3, 555–565; MR0494062 (58 #12996)]. Niblo showed that they are double coset separable [*J. Pure Appl. Algebra* 78 (1992), no. 1, 77–84; MR1154898 (92m:20019)]. Ribes, Segal and Zalesskii showed that if $G = \pi_1(M)$ and M has nontrivial boundary, then G is conjugacy separable [*J. London Math. Soc.* (2) 57 (1998), no. 3, 609–628; MR1659845 (99m:20053)]. The authors proved that if the base surface of M is orientable, then G is conjugacy separable [*J. Algebra* 285 (2005), no. 2, 481–507; MR2125449 (2005k:20072)]. The purpose of this paper is to complete the picture to the case where the base surface of M is not orientable.

The proof of the main result uses the classification of Seifert 3-manifold groups and makes use of a technical characterization of conjugacy separability of an amalgamated product of two conjugacy separable groups with a \mathbb{Z}^2 -amalgamated subgroup which is developed in their earlier paper [*J. Algebra* 285 (2005), no. 2, 481–507; MR2125449 (2005k:20072)].

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