ON A CLASS OF IRREDUCIBLE REPRESENTATIONS OF THE BRAID GROUP B_n

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The present article studies the algebraic properties of a certain representation of the braid group due to Wada in [*Topology*, 31 (1992), 399–406; MR1167178 (94e:57014)]. These representations are given by actions of the braid group on the free group in a way which differs from the standard representation discovered by Artin. Wada discovered the following three representations:

(1) For each non–zero integer k, the standard braid generator σ_i takes $x_i \mapsto x_i^k x_{i+1} x_i^{-1}$, $x_{i+1} \mapsto x_i$, and fixes the other free generators.
(2) The generator σ_i takes $x_i \mapsto x_i x_{i+1}^{-1} x_i$, $x_{i+1} \mapsto x_i$, and fixes the other free generators.
(3) The generator σ_i takes $x_i \mapsto x_i^2 x_{i+1}$, $x_{i+1} \mapsto x_{i+1}^{-1} x_i^{-1} x_{i+1}$, and fixes the other free generators.

Linear representations are obtained using the standard Magnus representation, thus giving representations of braid groups into matrices over $\mathbb{Z}[t^{\pm 1}]$, the ring of Laurent polynomials in one variable. In earlier papers, the authors had proved irreducibility for representations of the first two types. In the article under review, they perform very concrete calculations to prove that for specializations of the form $t \mapsto z \in \mathbb{C}$, the third representation is irreducible if and only if t is not specialized to a square root of unity.

In the second part of the paper, they produce a certain hermitian matrix and show that the third representation is unitary with respect to that matrix.

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