Lecture 1: A drawing game

In this first lecture, we look a mathematical concept and try to place into the realm of mathematics. It is a topic which I stumbled upon when looking at graphs and which I still work on. The underlying construction is something which kids can already do, in order to produce pretty coloring shapes.

Draw a polygon onto a piece of paper. Lets assume we have drawn a pentagon. This polygon consists of $v = 5$ vertices, $e = 5$ edges and $f = 1$ faces.

We will refine the polygon as follows:

1) Mark the edge centers and face centers.

2) Connect the face centers with the old vertices.

3) Connect the face centers with the edge centers.

We have now $v = 11$ vertices and $e = 20$ edges and $f = 10$ faces.
We want to see what happens if we repeat this construction.

Now its your turn. Take a piece of paper, draw a polygon and refine it. Once you are done. Refine again.

For your information, what you played with here is called **Barycentric subdivision**. It has applications in computer graphics.

### Questions

1. Let’s see what we can count at the various stages. Count the number of triangles. What do you observe?

2. Look at the number $v - e + f$ adding the number of vertices and faces and subtracting the number of edges. What do you observe?

3. How does the number of neighbors change? For example, if a vertex has 4 neighbors, how many neighbors will it have in the next step?

4. What can you say about the size of the triangles? Especially, what happens with the area? Can you say something about the angles?